Attenborough School

Calculation Policy

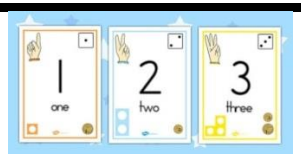
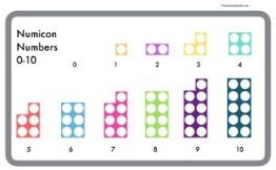
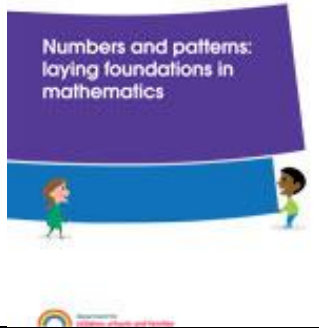
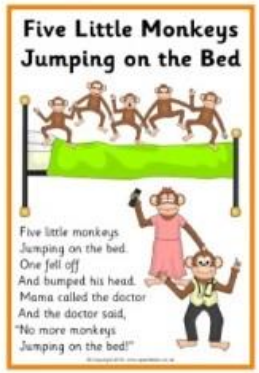
March 2018

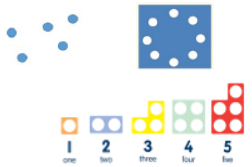
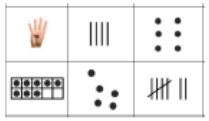
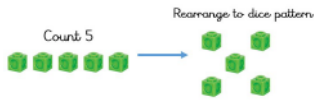
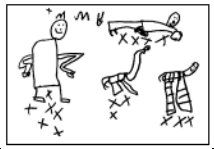
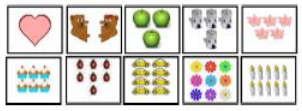
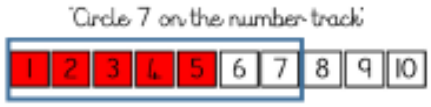
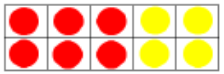
Contents:

Progression in Multiplication skills

Progression in Division skills

	Objective	Enabling Environments	Concrete, Pictorial, Abstract
	Role of the adult	Provision of adult led and child led learning that opens up possibilities. Adults watch for learning opportunities and intervene sensitively where a teachable moment occurs. Adult modelling thinking and vocabulary and posing *genuine* questions in a meaningful context.	
By 24 months	*Matches one object with another object or picture during play. *Participates in number rhymes and action games.	• Role play settings in the home corner • Shadowing to support find-use-return • Puppets and props to support rhymes (and CD players) • Picture lotto • Number stories and rhymes available in areas	    
By 30 months	*Organises a set of objects in a group. *Plays hide and seek and know objects exist even when they are out of sight. *Is beginning to compare quantities using vocabulary such as more or a lot.*	• Collections such as: a) Natural materials b) Buttons c) Compare bear • Small world materials • Fabric • Feely box/bag • Snack items that allow for comparison e.g You have a lot of raisins, I have more grapes than…."	 
By 36 months	*Use number names in play. *Show an interest in numbers in the environment. *Beginning to represent numbers in a variety of ways including on paper.*	• Target/scoring games indoors and out • Marking number of children in their group • Number hunts • Snack numbers • Telephone numbers, maps and messages in role play • Representing rhymes (e.g. 5 speckled frogs, 5 wonky bicycles)	   

	Objective	Enabling Environments	Concrete, Pictorial, Abstract
By 42 months	*Makes comparison between different quantities, *saying when they have the same number.* *Can count alongside actions in games, rhymes and songs. *Uses some numbers accurately in play.* *Can separate a group of up to 4 objects in a range of ways e.g 3 + 2, 4 + 1. Recognises some numerals of personal significance.*	• 5/10 frames during registration • Comparing the number of children in/out during registration • Snack, Cooking • Number Songs, Stories and Rhymes sung as a group, on listening stations, posters available • Role play areas provide opportunities for using number names e.g. scales for cooking, telephones, matching items • Outdoor and indoor games for scoring • In number rhymes draw attention to the composition of the group • Use interesting pictures and objects that support composition of number • Number hunts, birthdays, number bongo games, numicon in dough, construction and availability of number lines.	      

	Objective	Enabling Environments	Concrete, Pictorial, Abstract
By 48 months	*Recites numbers in order to 10. *Uses graphic representations to record number explorations in pictures and mark making. *Can count up to 6 objects.* *Recognises numerals 1 – 5.* *Estimates how many objects they can see and checks by counting them (to 6).*	• Matching games • Number lines in various representations • Keeping score in football/games • Numbers on bikes with matching parking spaces • Recording in different ways e.g. tally, dots etc • How many boys/girls/teachers do you think are here today? • Role play – home corner, shop, café, bus etc • Number of children in class/snack menus/number of children in a particular area • Number rhymes, stories and games • Snap and pairs games	         
By 54 months	*Counts with 1:1 correspondence a set of up to 10 objects and recognises numerals to 10. *Finds totals by counting and combines groups of objects. *Counts irregular arrangements, actions or objects which cannot be moved.*	• IWB games – Espresso/Education City • Display numbers in a variety of arrangements and visual representations. • Exploring different ways to make numbers using numicon, dominoes, egg boxes. • Counting claps, sounds, actions • Purposeful use of number in the environment e.g. how many children can play in an area • Number lines and hundred squares, bundles of sticks • Counting strings and sticks • Combining towers of Duplo and other construction. • Counting the number of children in the line	           

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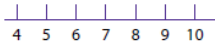
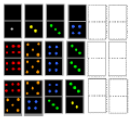
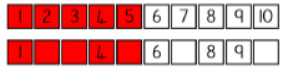
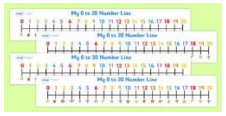
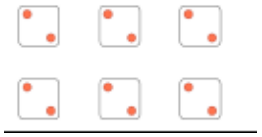
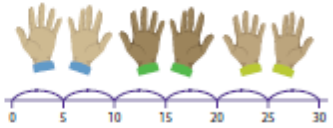
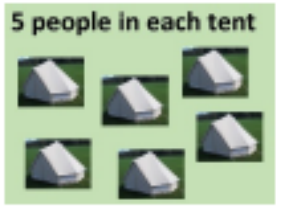
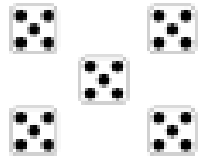
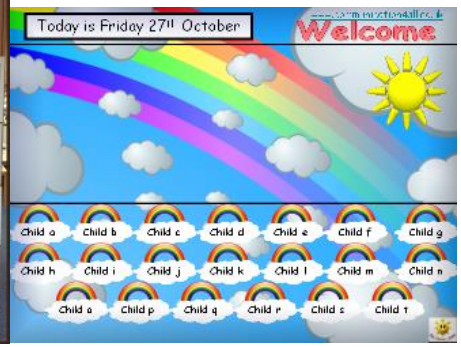
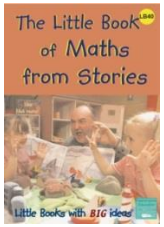
EYFS (FS2) - Calculation Policy

	Objective	Enabling Environments	Concrete, Pictorial, Abstract
By 60 months (ELG Emerging)	*Counts reliably with numbers from 0-10. Recognises and places in order and uses resources to say one more or one less than a given number. *Orders numbers 0 – 10 and understands the relationship between a group of objects and the corresponding number 0 – 10. Can add and subtract single digit numbers in their play by counting on or back to find the answer. *Compares groups of objects using language such as more or fewer or equal/the same.*	• Grab and count mats • Number tracks, lines, hundred squares • Numbers on cars and simple addition/subtraction on parking spaces • Remote control cars with numbers • Padlocks and keys with numeral and representation • Are there more boys/girls? • Number rhymes, stories games e.g. 10 green bottles • Recipes – snack & cooking • Subitizing games • IWB games: Education City & Espresso	             
By 66 months (ELG Expected)	*Counts reliably with numbers from 0 – 20, place them in order and say which number is one more or one less than a given number. *They solve problems including doubling, halving and sharing.	• Board games • Snack – sharing food fairly • IWB games – Espresso & Education City • Hundred squares with different patterns highlighted. • Number hunts • Role play – money, shops, buses, café etc • Cooking – weighing scales, recipe books. • Number stories and songs • Organising larger quantities into equal groups. • Place value cards for combining tens and units • Counting forwards and backwards from different starting points supported by number rhymes and number lines. • Small world resources	         

	Objective	Enabling Environments	Concrete, Pictorial, Abstract
67 months + (ELG Exceeding)	*Can estimate a number of objects and check quantities by counting up to 20. *Solve practical problems that involve combining groups of 2, 5 or 10 or sharing into equal groups.	• Counting pairs when lining up • Matching and counting socks • Snack items that promote sharing • Estimating quantities in play e.g. how many cars do you think we have? How could you check? • Counting coins to pay for amounts. • Board games • Printing repeated patterns with numicon • Small world resources	        

In the EYFS there are many mathematical opportunities that can be utilised throughout the course of daily routines

Registration	 
Management of children in spaces	The amount of aprons available, the chairs available at snack, bikes that are out, talking to the children about the space available – are there too many children? What can we do?
Snack	Subitising through the use of numeral, spots, items
Lunch time– setting the tables	How many knives and forks do we need? How many boys on the table? How many girls? How many altogether? What if another child sits at your table? What if one moves?
Story	We regularly sing and act out number rhymes that support counting and simple addition and subtraction. There are many good quality stories that support mathematical concepts e.g. http://www.mathsthroughstories.org/resources.html http://www.mathematicshed.com/maths-story-shed-2.html 

This cannot hope to cover all of the opportunities that come up during the course of the child's play; the role of the adult is therefore vital in listening, supporting and extending in context

Multiplication Tables – Y1-4

Year 1	Year 2	Year 3	Year 4
Counting in 2s, 5s & 10s	Times Table recall for: 2, 5, 10, 4* *4 times table are in the curriculum for Y3. They can be introduced in Y2 summer term and the link with the 2 times table can be taught.	Times Table recall for: 4, 3, 8, 6* *6 times table are in the curriculum for Y4. They can be introduced in Y3 summer term and the link with the 3 times table can be taught.	Times Table recall for: 6, 7, 9, 11, 12

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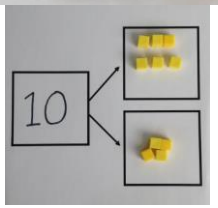
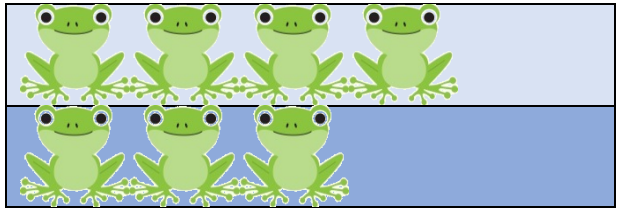
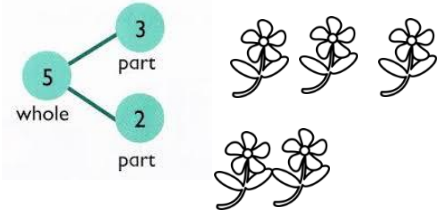
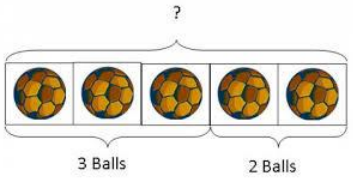
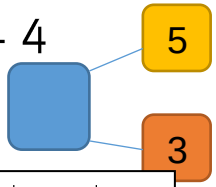
Calculation Policy

	Year 1	Year 2	Year 3	Year 4	Year 5
Addition	• Combining two sets to make a whole: part- whole model • Early commutative properties of addition. • Counting on from the largest number. • Breaking down numbers. • Breaking down numbers to calculate.	• Adding three single digits. • Adding 2, 2 digit numbers using a number line – **Numbers within a 10s boundary.** • .Adding 2, 2 digit numbers using a number line – **Numbers cross a 10s boundary.**	• Addition using the column method no regrouping – Numbers within a tens or hundreds boundary • Addition using the column method – regrouping and carrying digits.		

Subtraction	• Subtraction as less – Subtraction as taking away
	• Counting back in ones.
	• Subtraction as finding the difference.
	  ○ Bar Model
	  ○ Part, Part Whole
		• Subtracting to a 10, then continue.
		• Purposeful questioning to encourage reasoning.
		• Subtraction of 2 digit numbers – Part 1
		• Development of number line skills.
		• Subtraction of 2 digit numbers – Part 2
			• Subtraction of 2 digit numbers – Part 2
			• Subtracting using a column method.
			• Deepening understanding of subtraction using a column method.
			• Exchanging zero.		

Multiplication	• Counting in multiples • Doubling	• Repeated addition. • 2x table, 5x table & 10x table. • Arrays- showing commutative properties of multiplication • Breaking down arrays	• Deriving facts from known tables. • Multiplying by 10, 100 & 1000. • The Grid Method • Scaling, Ratio & Combinations - (context questions)	• Column multiplication - (short) *Two and three digits multiplied by a one digit number.*	• Column multiplication - (long) *Up to four digits multiplied by a two digit number.* - **Grid Method** • Column multiplication - (long) *Up to four digits multiplied by a two digit number.* - Formal Method

Division	• Division as sharing objects into equal groups
	• Division as grouping objects.
		• Division as grouping objects.
		• Contexts for division.
		• Division with arrays
			• Language of division
			• Deepening division concepts
			• Remainders with division
			• Early Written Methods
			• Number line
				• Early Written Methods
				  ○ *Short Division with manipulatives (1)*
				  ○ *Short Division with manipulatives (2)*
					• Long Division - *Chunking*
					• Long Division - *Traditional Method*
					• Long Division - *Larger numbers*

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two sets to make a whole: part- whole model	    Use cubes and other models to add two numbers together as a group. This could be presented in groups or as a bar.  E.g. How many frogs are there in two ponds?	  3 Balls 2 Balls  Use pictures to show two sets of numbers, this could be shown in a bar as shown with the footballs, moving towards a more abstract picture of the bar model.	4 + 3 = 7 10= 6 + 4  Calculations are shown in their abstract form as number sentences. Children have a secure understanding of what the symbols mean and represent and use informal strategies (from the concrete and pictorial) to calculate.

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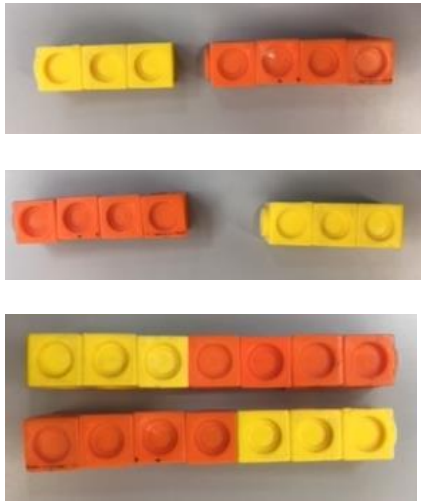
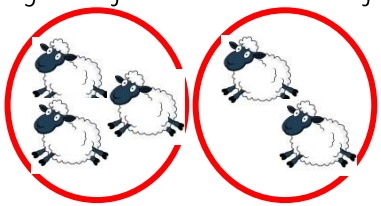
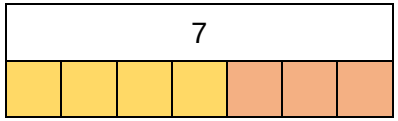
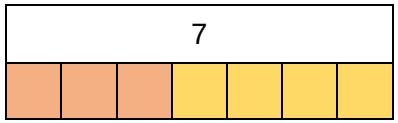
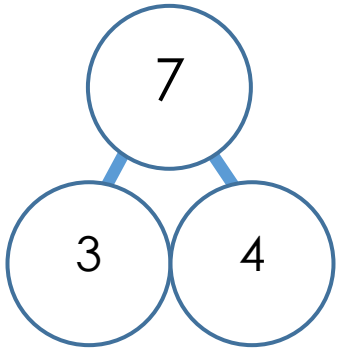
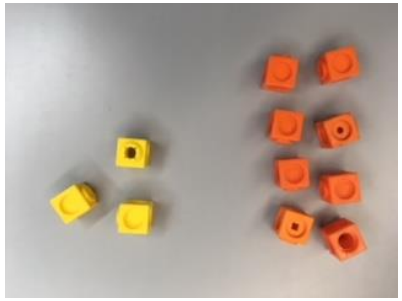
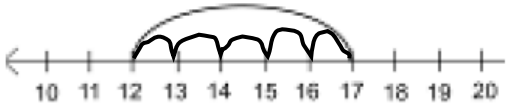
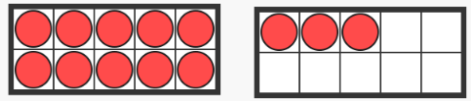
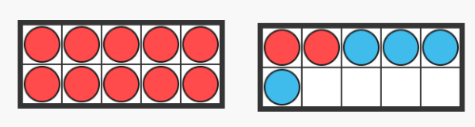
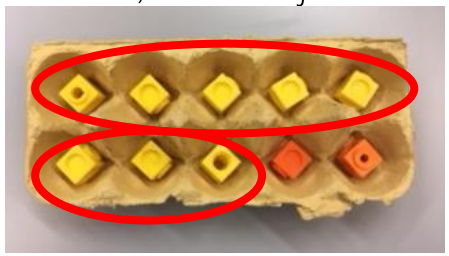
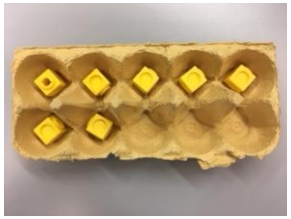
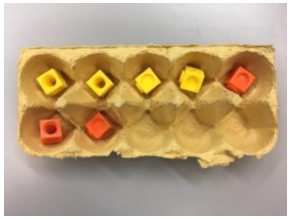
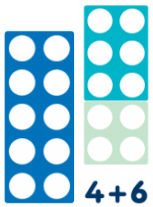
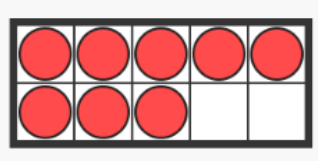
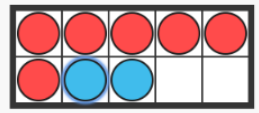
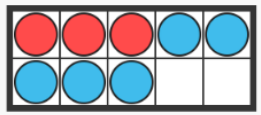
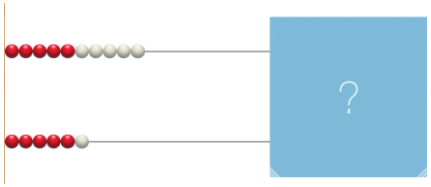
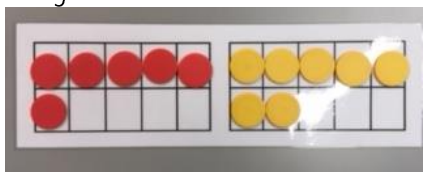
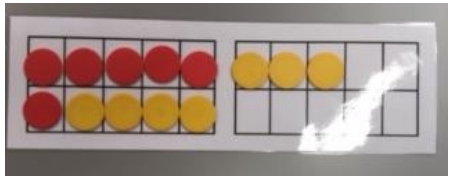
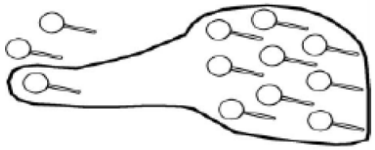
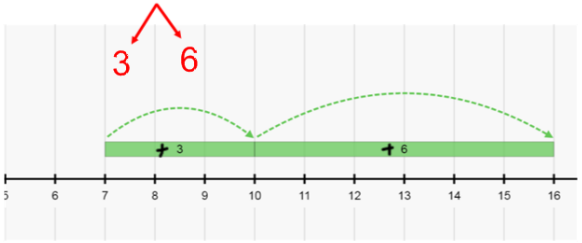
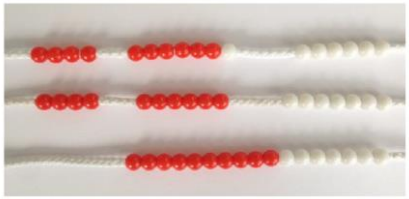
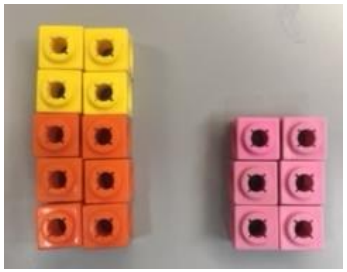
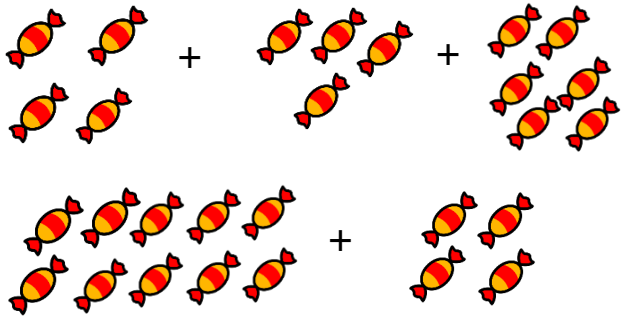
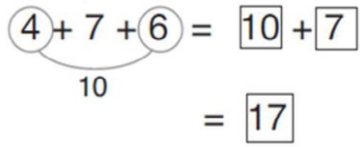
Addition - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Early commutative properties of addition.	 Children use cubes and other resources to understand that addition can be solved in any order. The order of the numbers doesn't affect the result. e.g. 3 + 4 = 7 4 + 3 = 7 This can be developed through varous models and contexts. e.g. Two fields with two sets of sheep. How many altogether?  3 + 2 = 2 + 3	  Bar models and part-part whole models further support this concept.  What number sentences can be made using these facts? • 3 + 4 = 7 • 4 + 3 = 7 • 7 – 4 = 3 • 7 – 3 = 4	12 + 8 = 20 8 + 12 = 20 The abstract number sentences can be shown with the numbers 'swapped' around. Children should have a secure understanding of this concept before using written addition sentences. Continuous work on number pairs will also show help develop this concept.

Objective and Strategies	Concrete	Pictorial	Abstract
Counting on from the largest number. *Physically with bead strings.* *Moving towards* *Structured number lines.*	Seeing 3 + 6 as 6 + 3. Hold 6 then count on from 6 in ones using multilink to support. Start with models that fit within a context.  Move towards more abstract models  Into a more efficient concrete resource for larger numbers. The bead string.  Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.  Using 10 frames, Seeing 13 by knowing there's 10 and 3. Holding 13 in the head and counting on to find the total to 20. Or  Holding 12 in your head and counting on to find the total shown in these 10 frames - 16	5 + 12 = 17 **Building on from commutative properties** knowing 12 + 5 will gain the same result. Place the **larger number** in your head and count on the smaller number to find your answer.

Objective and Strategies	Concrete	Pictorial	Abstract
Beginning to manipulate numbers – Developing number sense.			
Breaking down numbers. Using 10 frames to find pairs of numbers up to 10 & 20.	Exploring pairs of numbers that make 10. Use egg boxes as a concrete frame. Start to develop number sense. "How many yellow cubes" – 5 and 3 = 8. Build on from counting. Know there are 5, count on to find 8.  8 + 2 = 10 Develop this into finding how many more would make 10 as well as building on part-part whole concepts. 7 + ___ = 10  7 = 4 + 3  Use a variety of other resources to support the understanding of pairs to 10, pairs within 10, pairs to 20 and pairs within 20.   4+6	Using visual resources develop concept of number pairs up to 20.  How many more to make 10? How many ways can you show 8?  6+2  3+5  How many are hidden? How do you know?	Children are confident writing calculations for pairs of numbers. e.g. 5 + 5 = 10 6 + 5 = 10 Tackling missing number problems using knowledge of concrete and pictorial work. 10 = 2 + ___ How many ways can you split 6? 1 + 5 2 + 4 3 + 3

Objective and Strategies	Concrete	Pictorial	Abstract
Breaking down numbers to calculate. Regrouping to make 10.	Use 10 frames, rekenrek & numicon to see patterns of number to make 10 and 20 Working alongside understanding pairs of number, being able to see 6 + 7 in different ways. Physically manipulating the '7' using resources.  7 can be made up of 4 & 3. 6 + 4 makes 10 and then 3 is 13.  Numicon  7 + 8    8 can be partitioned into 3 & 5 7 + 3 = 10 10 + 5 = 15	Through the use of pictures or a structured number line. Regroup or partition the smaller number to make 10.  3 + 9 = or 7 + 9 = 	Building on visual strategies to be able to work mentally. 7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

Objective and Strategies	Concrete	Pictorial	Abstract
Adding three single digits	This can be done using various concrete resources to see patterns and pairs of numbers. Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit. 4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  3 + 4 + 7  6 + 4 + 6  3 + 7 = 10 and then + 4 = 14  6 + 4 = 10 and + 6 = 16 	Progress into using pictures to support this concept. Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	Children should be able to confidently find complements to 10 and will be able to use this when looking at mentally calculating number sentence like the one below.  Combine the two numbers that make 10 and then add on the remainder.

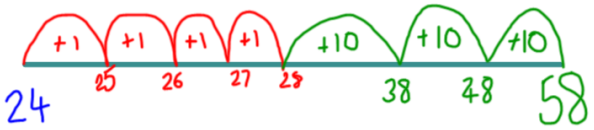
<table>
<tr><th>Objective and Strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Adding 2, 2 digit numbers.

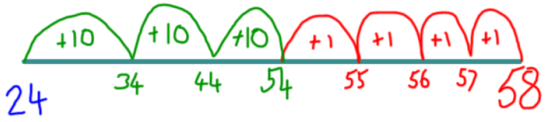
Using a number line – Numbers within a 10s boundary.</td>
<td>
Children build on knowledge of addition being the combining of two groups. Their understanding of the commutative law will support with counting on when combining as opposed to combining the two numbers and counting the whole amount from zero.

24 + 34 = would be calculated using dienes. The numbers don't cross over a 10s boundary to begin with.
<table>
<tr><td>Start by making both numbers.</td><td>  </td><td>Reiterate counting on strategies. Start with both numbers.</td></tr>
<tr><td>Add the ones together.</td><td>  </td><td>Count on the ones: 24,25,26,27,28</td></tr>
<tr><td>Add the tens to find the total.</td><td>  </td><td>Count on the tens: 28, 38, 48, 58</td></tr>
</table>
</td>
<td>
Progressing into the use of an unstructured number line.

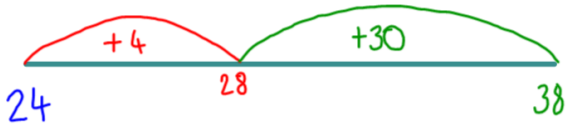
24 + 34 =

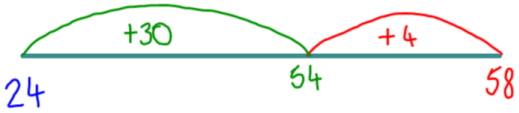
Counting in jumps of ones and tens.



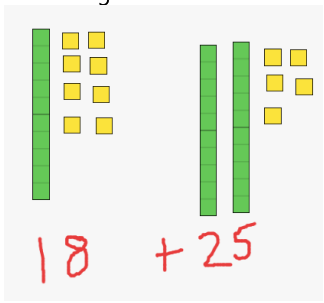
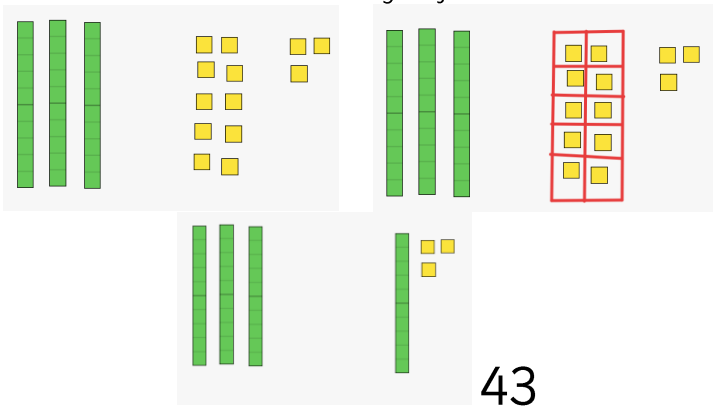
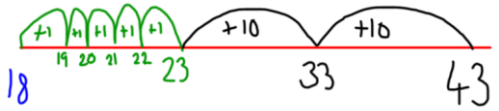
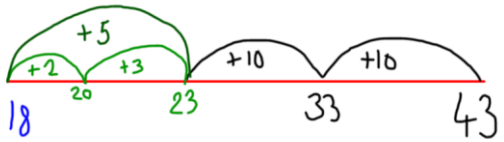


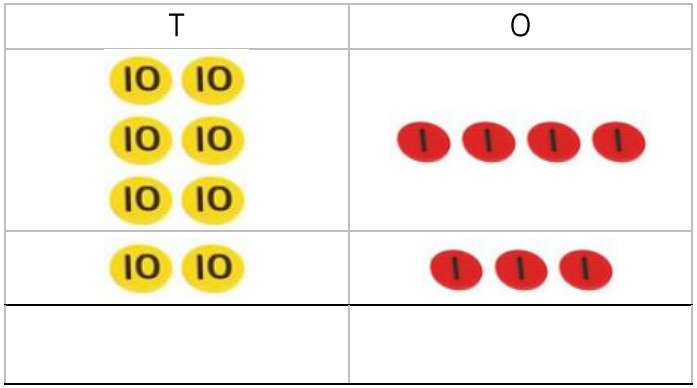
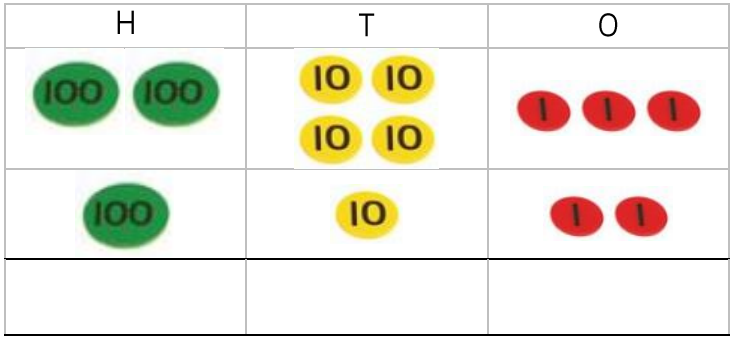
Confidently jumping multiple ones and tens in one step.





Children choose the method they're confident with and move towards the most efficient.
</td>
<td></td>
</tr>
</table>

Objective and Strategies	Concrete	Pictorial	Abstract
Adding 2, 2 digit numbers. **Using a number line – Numbers cross a 10s boundary.**	When children are confident adding two groups using dienes, number lines, one totalling more than 10 can be introduced. e.g. 18 + 25 =  Children count the ones together. They can spot that 5 + 8 = 13. Using 10 frames, the children identify the **10 within 13**. Practical resources will help the children see that 30 + 13 is the same as 30 + 10 + 3. Ten ones can be 'exchanged' for a 1 ten.  **Children must be able to identify the tens within the ones before moving onto column addition.** **e.g. 16 ones = 1 ten and 6 ones = 10 + 6**	The numberline continue to be embeded to add two numbers that cross a 10s boundary. Children can begin to apply their understanding of complements to 10 from using the dienes. e.g. 18 + 25 from jumping in ones and tens.  Progress to using complements to 10. This builds on the work from the concrete, using the 10 frames to see how many more will make 10. 18 + 2 = 20 20 + 3 = 23 23 + 20 = 43  	

Objective and Strategies	Concrete	Pictorial	Abstract
Adding using the column method **No regrouping**	When children are confident with place value & the number line method, they can begin to understand the organisation of the column method. **Using dienes then place value counters** 64 + 23 = 60 + 4 20 + 3  The calculation can be done practically, with the children moving the counters down when counting them all together. 243 + 112 =  		This can lead towards the abstract partitioned written method. 24 + 34 = 24 = 20 + 4 34 = 30 + 4 | | T | | O | | | | | 20 | + | 4 | | | \+ | 30 | + | 4 | | | | 50 | + | 8 | = | 58 | And then into the formal written method Expanded | | T | O | | | | 2 | 4 | | \+ | 3 | 4 | | | | 8 | 4 + 4 | | 5 | 0 | 20 + 30 | | 5 | 8 | | Compact | | T | O | | | 2 | 4 | \+ | 3 | 4 | | 5 | 8 |

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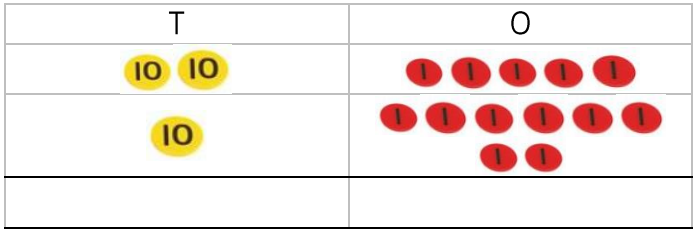
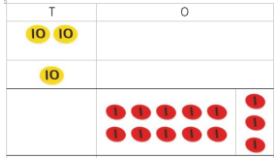
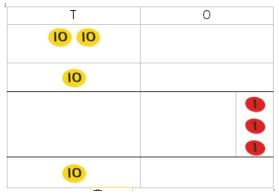
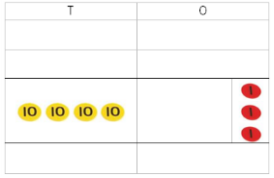
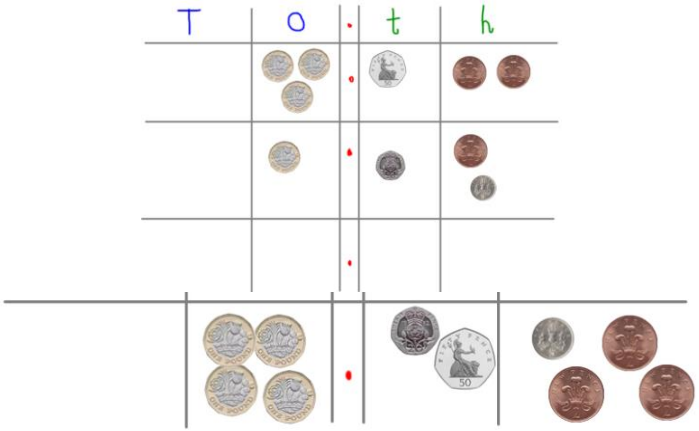
Addition - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Adding using the column method **Regrouping**	The children use their knowledge of place value to understand that when adding ones with a total more than 10, 10 ones can be exchanged for 1 ten. **Using dienes then place value counters** 25 + 18  When calculating the ones.  Notice that 5 + 8 = 13  13 can be made with 10 & 3. Carry the 10 into the tens column.  Count the tens. 20 + 10 = 30 30 + the one carried 10 = 40. The total is 43 The principle can be applied to larger 3 digit & 4 digit numbers.		This concrete application can lead towards the abstract partitioned written method. 18 + 25 = 18 = 10 + 8 25 = 20 + 5 T O 10 + 8 + 20 + 5 30 + 13 = 43 30 + 10 + 3 = 43 And then into the formal written method Expanded T O 1 8 + 2 5 1 3 8+5 3 0 10+20 4 3 3+0 then 10+30 Compact T O 1 8 + 2 5 4 3 1

Objective and Strategies	Concrete	Pictorial	Abstract
Adding money using column addition.	The children apply their understanding of decimal place value towards money, knowing that 10p represents 1/10 of a pound and 1p represents 1/100 of a pound. £1 is a whole amount. Money can be used to illustrate this concept and the method can be explored practically Knowledge of place value is essential as coins can't always be manipulated as place value counters can.  £4 + 70p + 11p = £4.81		This concrete application can lead towards the abstract written method. Expanded Compact

	T	O	.	t	h	th
		3	.	5	4	
+		1	.	2	7	
			.			

	T	O	.	t	h	
		3	.	5	4	
+		1	.	2	7	
			.	1	1	4p+7p
			.	7	0	50p+20p
		4	.	0	0	£3+£1
		4	.	8	1	

	T	O	.	t	h
		3	.	5	4
+		1	.	2	7
		4	.	8	1
				~~1~~	

Objective and Strategies	Concrete	Pictorial	Abstract

Mastery of Addition

When children have mastered the strategies for addition, activites and challenges (such as these examples) can be used to deepen and develop their understadning.

<u>Adding decimals.</u>

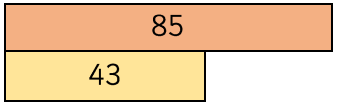
When children are secure with whole number column addition, decimals can be introduced to test their understadning.
E.g. 1.6 is 1 and 6 tenths = 1.60

	H	T	O	.	t	h	th
		2	6	.	5	1	
+			2	.	8		
		2	9	.	3	1	
			1				

They can apply their understanding of regrouping towards this, carrying the one whole *(5 tenths + 8 tenths = 13tenths or 1 whole and 3 tenths)* over into the ones column. Decimal place value counters can also help visualise this.

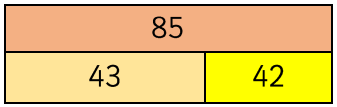
<u>Inverse relationship.</u>

Bar modelling can be used to show the inverse. E.g. 43 + ___ = 85



Children can use their knowledge of subtraction to identify 85 – 43 will find the missing value.

	H	T	O
		8	5
-		4	3
		4	2



<u>Most efficient method – Reasoning.</u>

How would you solve 546 + 298?

Using and applying number sense to calculations such as the one above.

Is a column method needed? Can you round 298 to 300 and then adjust by subtracting 2.

e.g. 546 + 300 = 846.

846 – 2 = 844

How would you solve 654 – 342?

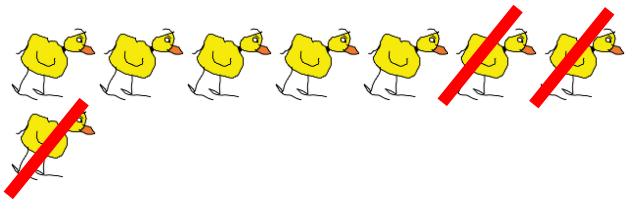
The number doesn't involve regrouping and doesn't cross a boundary. Can this be solved mentally?

<u>Missing digits.</u>

Children strengthen their understanding of the column method with missing digit problems. They require a secure understanding of regrouping. They understand that regouping of 10 ones occurs when there are more than 10 ones within the ones total.

	H	T	O	
		4		
+	3		7	
	8	7	2	
		1		

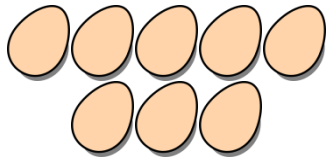
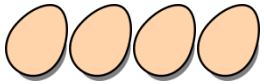
Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Subtraction as less – Subtraction as taking away	Models and examples used may fit within a context. e.g. There are 6 bears at a picnic and 2 go home early. How many are left? Start with 6.  Physically count back two and move them away. 	As children understand subtraction as taking away, they can draw their own images and cross out the ones they are removing. Contexts are still used to secure the children's understanding of this concept. e.g. 8 ducks are in a pond. Three fly away, how many are left? 8 – 3= 5 "8 (*1 step back)* 7 (*2 steps back)* 6 (*3 steps back)* 5 and my answer." 	Children are exposed to the abstract written calculation. 8 – 3 = 5 They are taught that the largest number represents the quantity you begin with and that the second number represents the number being taken away (counted back).

<table>
<tr><th>Objective and Strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Counting back in ones.</td>
<td>
Children will count back in ones. They will build on their understanding of numbers within 20 and can count back through 10 in ones confidently.

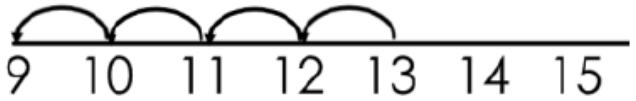
• 12 eggs and Mum uses 4 to make a cake. How many would be left? Make 12 eggs in total then take the 4 away by counting back as you do so.

12, 11, 10, 9, 8

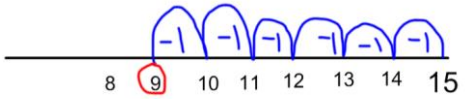
<table>
<tr><th>Realistic model</th><th>Alternative model</th></tr>
<tr><td>  </td><td>  </td></tr>
<tr><td>  </td><td>  </td></tr>
</table>
• Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

13 – 4 

• Use counters and move them away from the group as you take them away counting backwards as you go. 
</td>
<td>
Counting back in ones will progress from using concrete objects, drawing images and crossing out to the visual number line or number track

13 – 4 = 9
“13 (1 step back) 12 (2 steps back) 11 (3 steps back) 10 (four steps back) 9 and my answer.”


Start at the bigger number and count back the smaller number showing the jumps on the number line.

15 - 6 =


</td>
<td>
Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Children become fluent in backwards number word sequences. E.g. 10, 9, 8, 7, 6

Or

18, 17, 16, 15
</td>
</tr>
</table>

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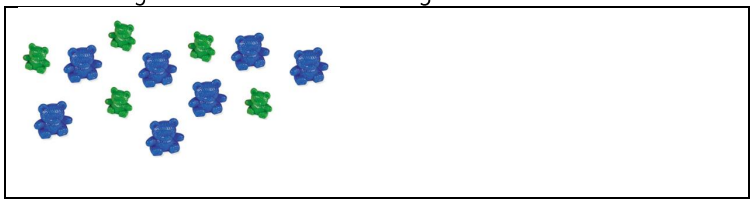
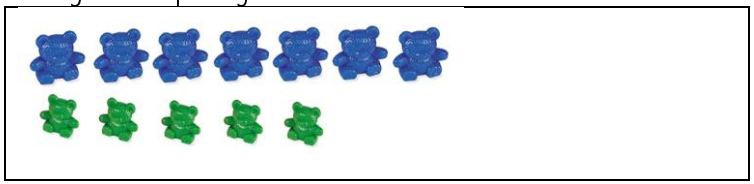
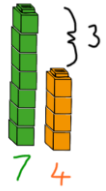
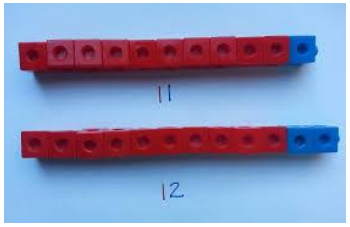
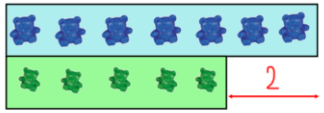
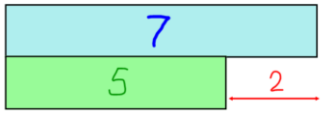
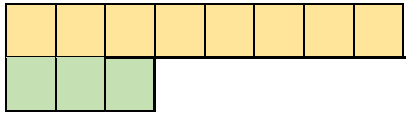
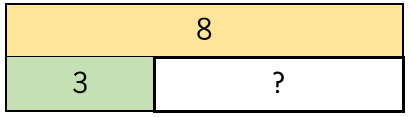
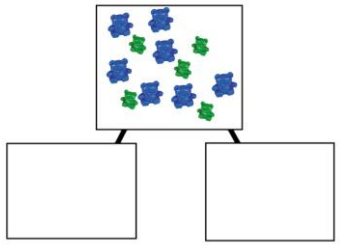
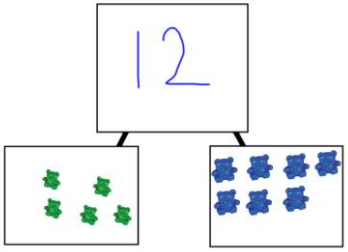
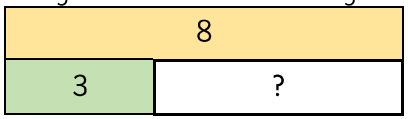
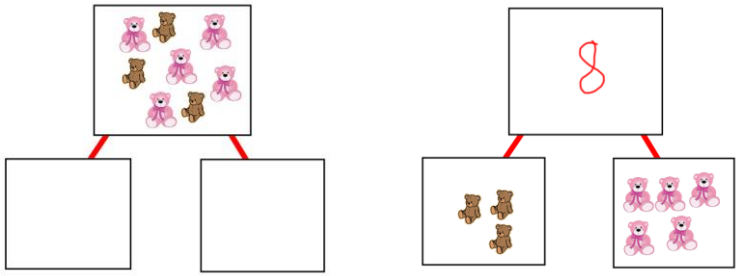
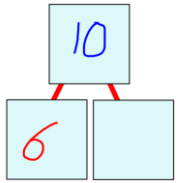
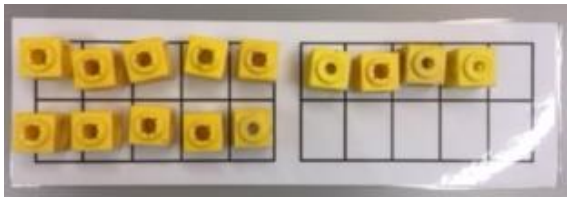
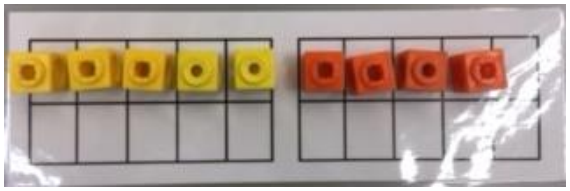
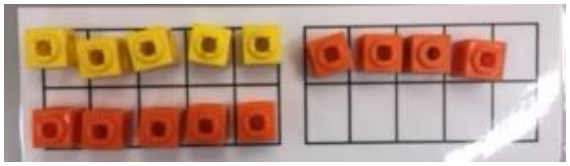
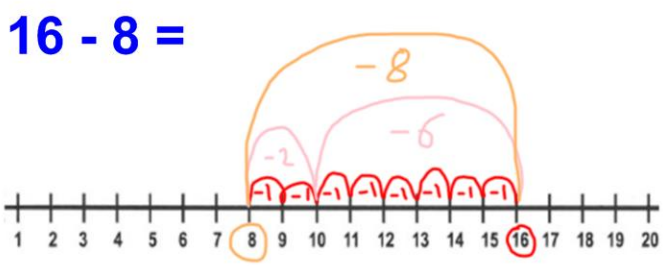
Subtraction - Calculation Policy

Objective and Strategies		Concrete	Pictorial	Abstract
Subtraction as finding the difference.	***Bar Model***	Using resources to look at two different sets of objects How many more blue bears than green bears?  Children can line up the bears to see the difference. By lining them up they can see there are two more blue bears.  Using multilink. Make two towers. How many are in your tower? How many are in your friends? How many more are there? What is the difference?   Numicon can also show this.  **8 – 5 = 3**	**Introducing the bar model early is essential** The bar shows subtraction as difference and can help to secure the relationship between addition and subtraction.   The bears become a bar → 7 – 5 = 2 or the **difference** between 5 and 7 is 2. Children are taught to look for the addition within subtraction. 7 - 5 = ☐ 5 + ☐ = 7 Subtraction as taking away would show 7 – 5 = 2. We want the children to also be able to think *'how many more from 5 will make 7'* and find the difference. Contexts can help introduce this. *Jack's tower is 8 blocks high but Sarah's is only 3. What is the difference? How much taller is Jack's?*   8 – 3 = 5	Difference needs to be visual and practical at this stage. With practice, children will recognise that 8 – 5 can be solved by finding the difference by starting at 5 and counting on.

Objective and Strategies		Concrete	Pictorial	Abstract
Subtraction as finding the difference.	***Part, Part Whole***	Using objects in the same way as the bar model. The link to addition can be visualised and developed. How many more blue bears than green bears?  How many bears altogether?  What if you took away the green bears? How many would be left?  If 12 is the whole and 5 is one of the parts, what is the other part?	Pictorial representations of the part part whole can be used alongside the bar model images.  Can also be seen visually  	The part, part whole can be shown using the digit representations of the numbers. 

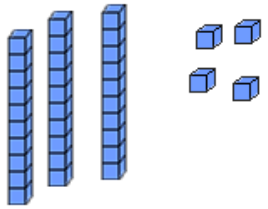
Objective and Strategies	Concrete	Pictorial	Abstract
Subtracting to a 10, then continue.	With the use of 10 frames, children are introduced to calculations that may cross a 10s barrier **14 – 9 =** The largest number is made.  The children have an awareness of how numbers can be partitioned. They can see that 14 is 10 + 4 so subtracting the 4 would make 10. 9 can be made up of 4 + 5  Changing the colour of the cubes can visualise the 14 and the 9 within this.  When 9 is then subtracted. We are left with 5.	This is developed using a structured number line. Start at the largest number.  Start at 16. Take away 6 to reach 10. Then take away the remaining 2 so you have taken away 8 altogether. You have reached your answer.	This concept can be developed mentally through secure understanding of the visuals. 16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

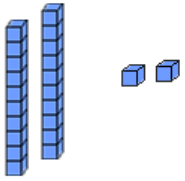
<table>
<tr><th>Objective and Strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td colspan="4">Purposeful Questioning to apply the skills above.
E.g. Rank by difficulty / mental or method?</td></tr>
<tr><td>9 – 3 =

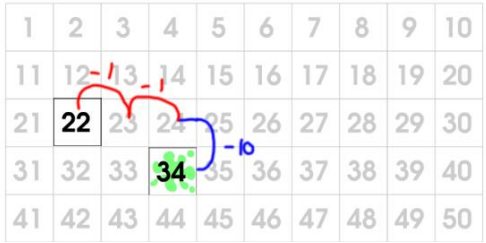
15 – 12 =

16 – 9 =</td><td colspan="3">A counting back strategy can be used here. Children discuss their reasoning for this.
Children should be able to see the closeness of these numbers and realise that finding the different will be more efficient
Counting back or finding the difference might take time. Subtracting to 10 then continuing would be more efficient. I know 9 = 6 + 3.
16 – 6 = 10
10 – 3 = 7</td></tr>
<tr><td>Subtraction of 2 digit numbers – Part 1</td><td>Children can contextualise subtraction, understanding the largest number is the quantity that exists. They know that subtraction cannot be done in any order (unlike addition) and must always start with the first largest number.

Calculations such as 34 – 12 can be solved using dienes.

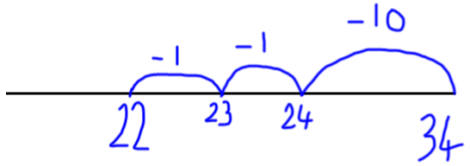
 
The whole amount can be made and then 12 can be physically taken away.

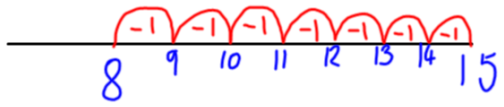
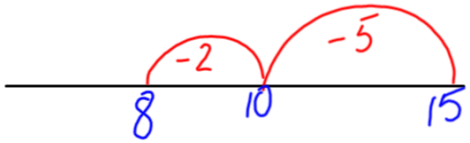
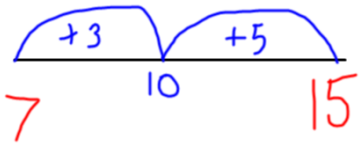
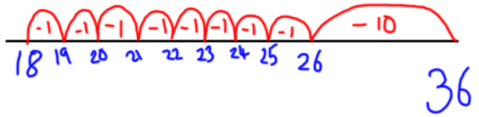
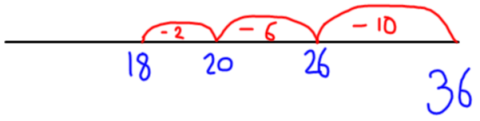
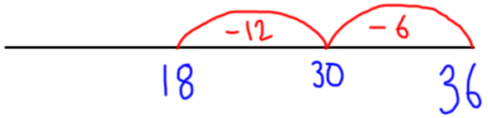
 
We are left with 22.</td><td>The 100 square is then used to support this visually.

34 – 12 = 22
 

34 – 10 = 24 then 24 – 2 = 22

This can be built upon using a blank number line.

 
</td><td></td></tr>
</table>

Objective and Strategies	Concrete	Pictorial	Abstract
Development of number line skills.		Numberline strategies become more efficient. 15 - 7 =  Progresses to: 15 - 7 =  The inverse is explored, with this numberline showing the difference. 7 + 8 = 15 so 15 – 7 = 8. 15 - 7 =  	Larger number can be subtracted when strategies become more efficient. 36 - 18 =  36 - 18 =  Larger jumps can happen as the children become more confident with this method. 36 - 18 =  Conversations can happen about which is the most efficient? Each method gains the same answer so are all good methods, but which one will reach the answer in the quickest way?	

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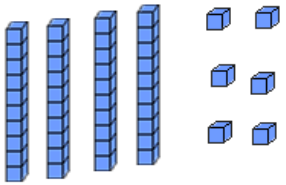
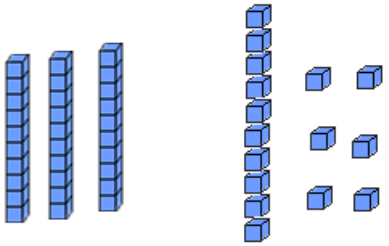
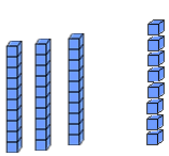
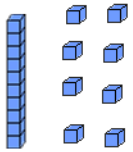
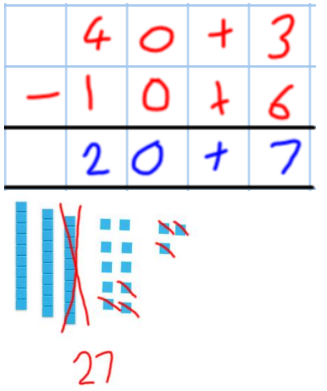
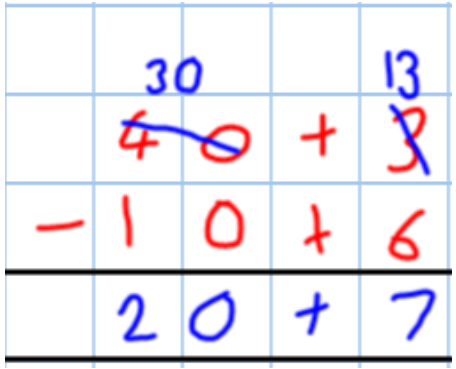
Subtraction - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Subtraction of 2 digit numbers – Part 2	46 – 28 will be difficult at first to calculate using dienes.  Through investigation, children will realise that you can't subtract 8 ones from 6 ones. They are taught and should see how to partition and regroup 46 in a different way. 46 = 40 + 6 but also 46 = 30 + 16  Now the 8 ones can be subtracted from 16 ones.  and 2 tens (20) can be subtracted giving us our final total of 18  The concept of subtraction involving regrouping should be expolored through practical resources until the concept it fully embeded.		When children have a secure understanding of this method, they can begin to set their work out as follows.  The method is worked on alongside the dienes. This progresses to the abstract when children fully understand. 

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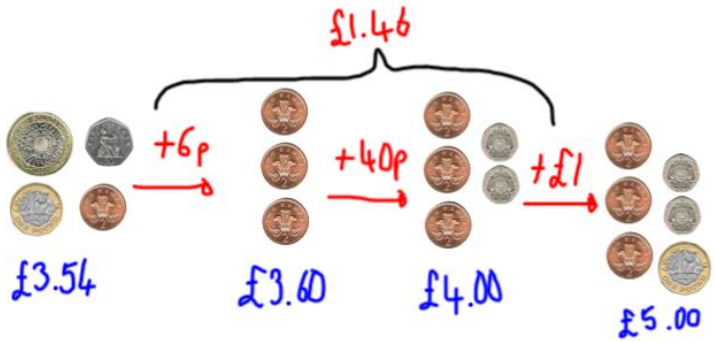
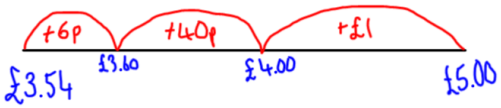
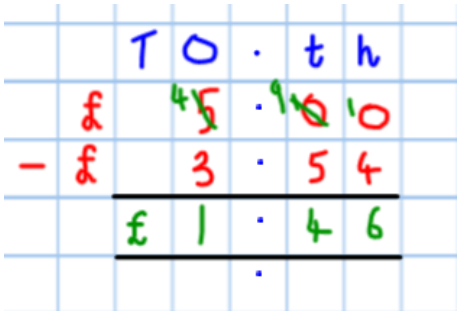
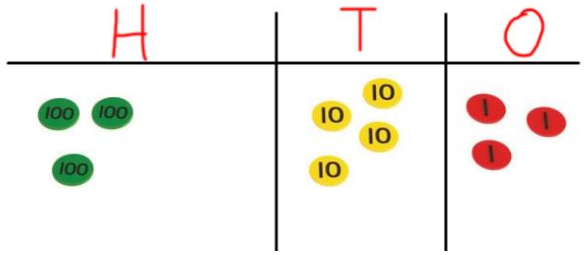
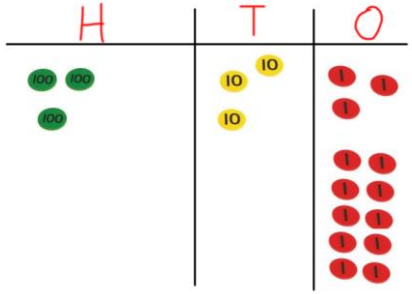
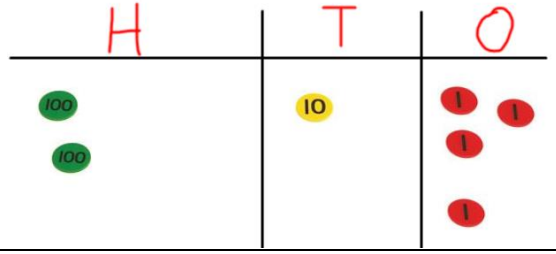
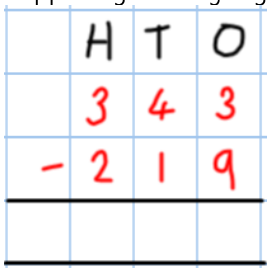
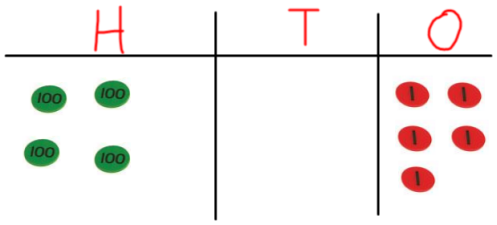
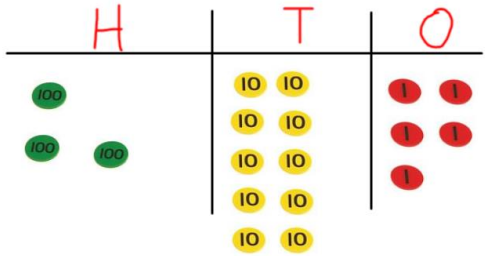
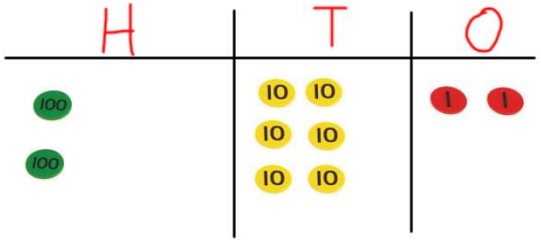
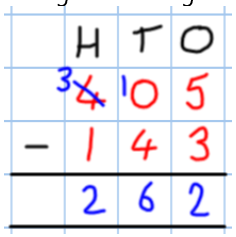
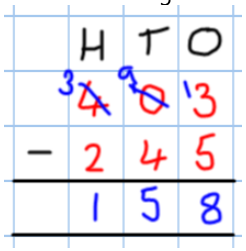
Subtraction - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Subtracting money to give change in practical contexts	Children will be familiar with coins. The concept that an amount of money (e.g. £5.00) can be made with different amounts of coins.  Children are taught to use difference to find change. Building on from their undestanding of difference in KS1. £5.00 - £3.54 can be seen as '*how much more needs to be added to £3.54 to get to £5.00?*' '*What is the difference between £3.54 and £5.00*'.  Children can use coins to count up to make the amount paid with.	Bar modelling can be used along side the concrete to explore problems and contexts given. E.g. Samantha buys a toy for £3.54. She pays with a £5 note. How much change will she receive? £5.00 £3.54 The bar model supports the concrete (left) and the bar can be completed with the difference. £5.00 £3.54 | £1.46 Numberlines can also be used to show difference using money.  	The children may progressto using a formal method. This will be developed as the children become fluent with column subtraction and decimal numbers in Y4 and USK2. 

Objective and Strategies	Concrete	Pictorial	Abstract
Subtracting using a column method	Dienes (as above) can be used to emphasise the place value of each digit. Place value counters can also be used to develop fluency with the column method. 343 – 129 =  300 + 40 + 3 = 300 + 30 + 13  The 129 can now be subtracted giving the answer of **214**  		Children can spot where regrouping is required and adjust the calculation accordingly. They have an understanding of place value and will know what is happening as they regroup.  

Objective and Strategies	Concrete	Pictorial	Abstract
Deepening understanding of subtraction using a column method **Exchanging zero**	Zero as a place holder is explored and children apply their understanding of place value and the column method towards calculations such as: 405 – 143  400 + 5 becomes 300 + 100 + 5  143 can now be subtracted – leaving a total of 262.  More advanced calculations such as 306 – 147 involving exchanging into the tens and then into the ones will test the children's fluency with this method.		Develop understanding of the layout along side using concrete resources.  Calculations with multiple regrouping will test children's understanding of the method.  Regrouping from a zero will further develop the children's fluency and understanding. 

Objective and Strategies	Concrete	Pictorial	Abstract

Mastery of Subtraction

When children have mastered the strategies for subtraction, activites and challenges (such as these examples) can be used to deepen and develop their understanding.

<u>Adding decimals.</u>

When children are secure with whole number column subtraction, decimals can be introduced to test their understanding.
E.g. 36.5 is 36 wholes and 5 tenths. 5 tenths is = 50 hundreths

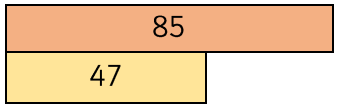
	H	T	O	.	t	h	th
		3	6	.	5		
-			2	.	8	2	
		2	9	.	3	1	

	H	T	O	.	t	h	th
		3	5 6	.	14 5	10	
-			2	.	8	2	
		3	3	.	6	8	

They can apply their understanding of exchanging towards this and place value couters can be used.

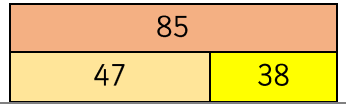
<u>Inverse relationship.</u>

Bar modelling can be used to show the inverse. E.g. 47 + ___ = 85



Children can use their knowledge of subtraction to identify 85 – 47 will find the missing value.

	H	T	O
		7 8	15
-		4	7
		3	8



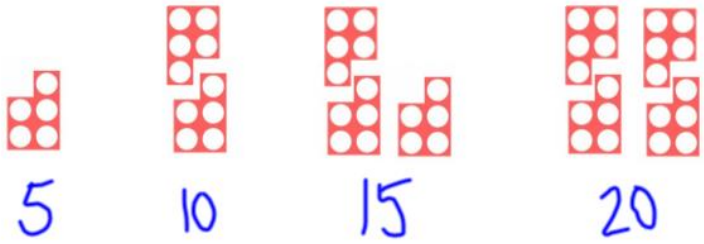
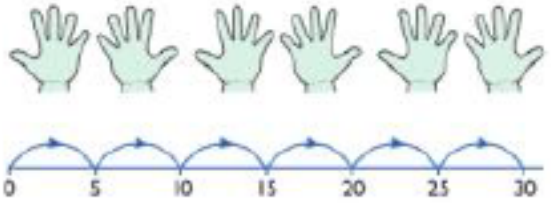
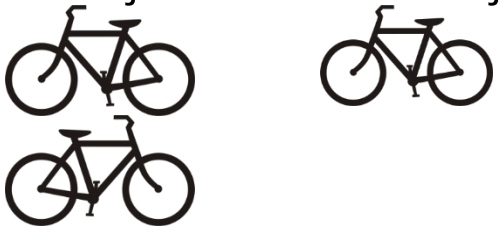
<u>Missing digits.</u>

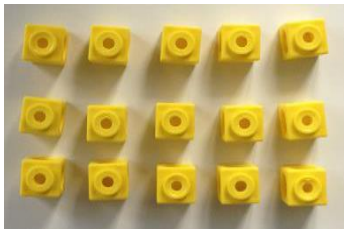
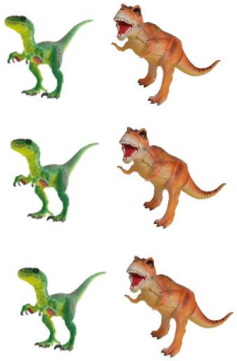
Children strengthen their understanding of the column method with missing digit problems. They require a secure understanding of exchanging.

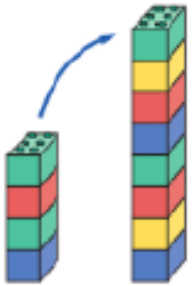
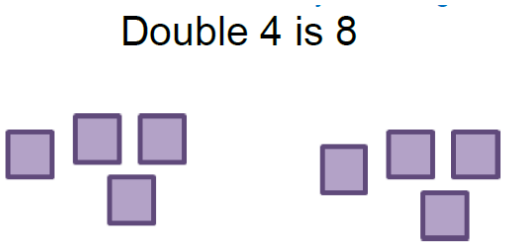
They will be able to identify that ___ - 20 = 80 must mean the missing value was 100, an exchange took place turning the original 0 into 10 tens = 100.

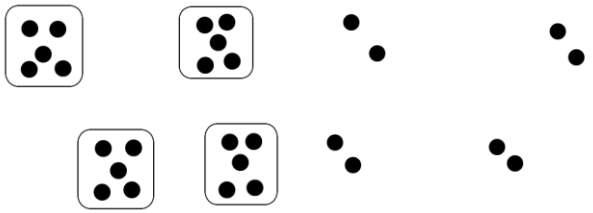
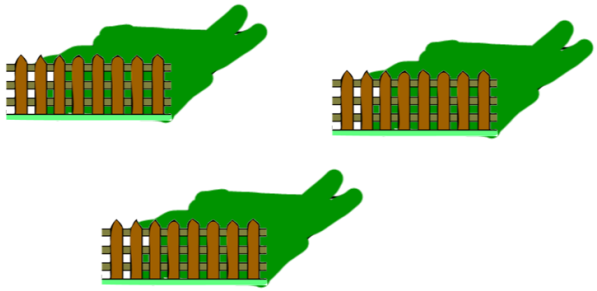
	H	T	O
			5
-	1	2	
	1	8	2

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Counting in multiples	Children experience multiplication as repeated addition when counting in multiples of 2, 5 and 10. Relationships with the real world are linked in. *E.g. the amount of wheels on 4 tricycles – 4 lots of 3 wheels.*  Multilink can be used to support this.  The bead string shows how counting in multiples increases the quantity e.g. 4 steps of 5: 5 + 5 + 5 + 5 = 20 5, 10, 15, 20 A range of concrete objects in equal groups are used to support this concept 	Pictorial representaton of equal groups develops children's understadning, allowing them to rely less on counting physical objects.  Real world examples can also be modelled through images. **How many socks are there in 6 pairs?**  **How many wheels are there on 4 bicycles?** 	Children are able to count in 2s, 5s and 10s orally without visual or concrete support. They can write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30 10, 20, 30, 40, 50

Objective and Strategies	Concrete	Pictorial	Abstract
Using an array. Using the term '**groups of**'	As children are taught to count in multiples, teachers may visualise this. This will be introduced formally in Y2.  Questions may be asked such as: 'How many **groups of** 2s can you see?' 'How many **times** are you going to count in 2s to find how many are there?' These questions will lead onto the concept of repeated addition that is taught in Y2. **2, 4, 6, 8**  Other arrays may be shown to represent counting in 5s and 10s. Questions like – 'How many are here?' may be used. **5, 10, 15** When counting totals, children may be encouraged to group counters, dienes and other objects into an array of 2s or 5s to help count the total.	Arrays may be shown visually to ask how many.  How many 2s are there? 3. **2, 4, 6**	

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use concrete resourses to explore the effect of doubling a number.  double 4 is 8 $4 \times 2 = 8$	Drawing images also shows the effect of doubling. Children begin to develop their understanding of doubling meaning two groups of equal quantity. 	Children can double numbers fluently up to 10 mentally using their understanding from working practically and visually. Double 3 = 6 Double 7 = 14 etc. They might begin to work with two digit numbers e.g. 13. Double 10 = 20 Double 3 = 6.

Objective and Strategies	Concrete	Pictorial	Abstract
Repeated addition.	Using concrete resources with equal groups, children begin to understand that counting them all together would be adding. 2 + 2 + 2 + 2 = **8**  Use bears to represent the following: 5 + 5 + 5 = 	Dot patterns can be shown to the children. How many are there? How can you count these?  Through discussion and exploration, the children can realise that there are 5 four times or 2 four times. Contextual questions can help children with the concept of repeatedly adding / counting in multiples. A farmer has 3 fields. In each field he keeps 5 sheep. How many sheep are there together? 	Children can see a question such as: 2 + 2 + 2 + 2 = and know that they are counting 2, 4 times = 8

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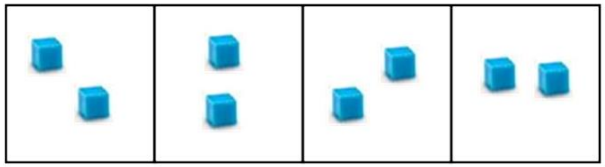
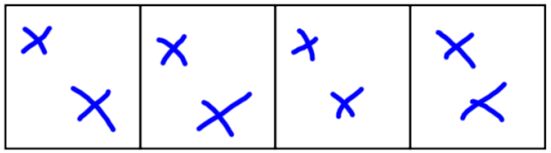
Multiplication - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
2x table, 5x table, 10x table & 4x table	Children use a variety of concrete objects to become fluent with the 2x, 5x and 10x tables. The 4x table can be introduced in the **summer term** and the link with the 2s can be taught in preparation for Y3. How many 2s?   How many pairs of socks? How many socks? They represent groups of 2, 5 & 10 using dienes / multilink / numicon  They are encouraged to look for examples of 'multiples' in every day life. How many arms do three children have? How many eyes do three people have. There are four cars. How many pairs of headlights?  	Children begin to draw group of objects. They use squares joined together to distinguish between multiplication being multiple lots of and division (sharing circles) being the quantity in one group once the whole amount has been shared.  They begin to make arrays to show groups of 2, 5 & 10.  Bar models can be used to represent multiplication. 12 2 2 2 2 2 2	Children are able to answer multiplcation statements fluently. 5 x 2 10 x 4 6 x 2 8 x 5 10 x 2 They understand and link the language of multiplication: Multiplied by – equal groups of – times by

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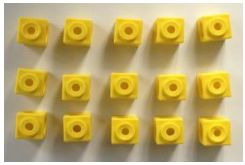
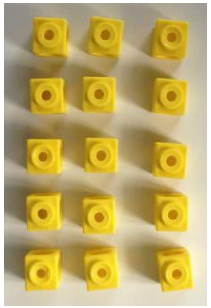
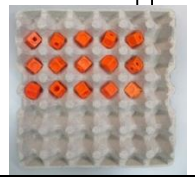
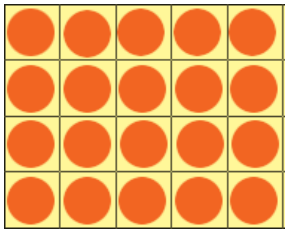
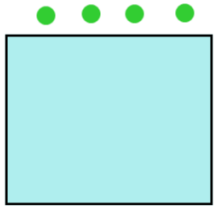
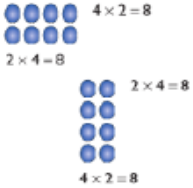
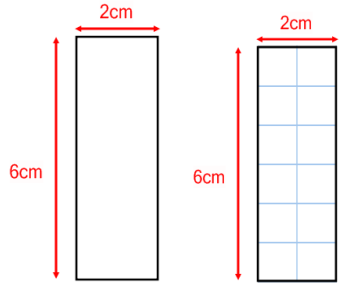
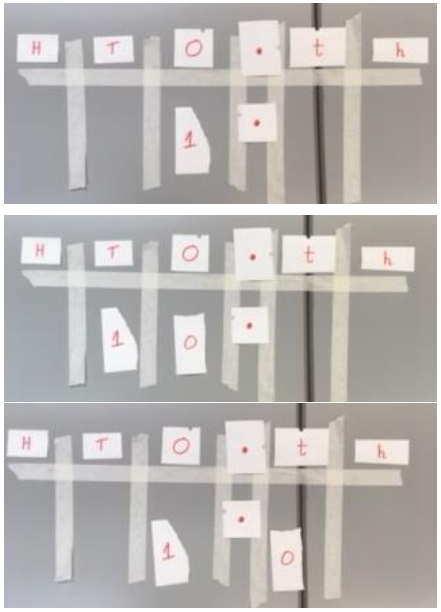
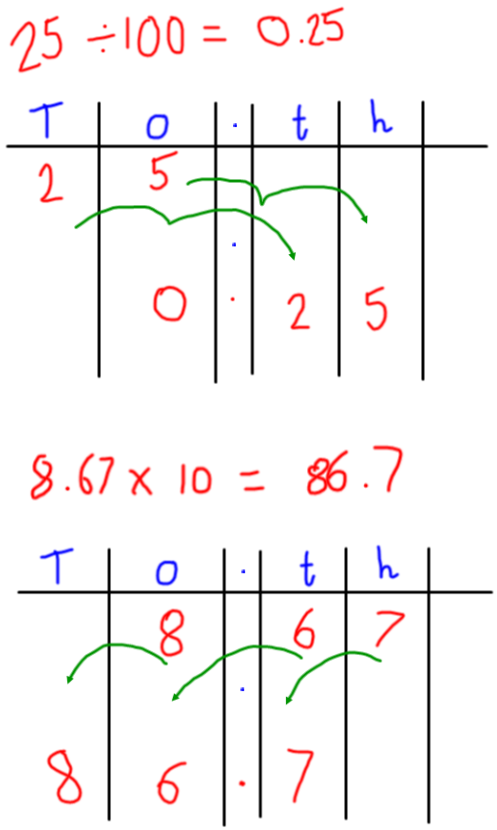
Multiplication - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Arrays- showing commutative properties of multiplication	Children begin to use arrays both informally and formally. Informally – How many rectangles can you make with 12 cubes? How many are in each row? Can you break your rectangle down into rows with equal amounts?  Formally –3 rows of 5. How many altogether? What is the calculation 3 x 5  Turn this around to see that 3 x 5 is actually the same value as 5 x 3  An egg box can also be used to support this. 	Show the array visually. How many in total?  Children may count in ones. Encourage them to count in 5s. 4 rows of 5 = 5, 10, 15, 20. Show an array with a selection covered. Embed the concrete understanding of how an array works. There are 5 rows and 4 in each row, how many are there altogether?  There are 5 rows and 4 dots in each row. How many dots are there? Draw arrays in different rotations to find **commutative** multiplication sentences.  	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15 Arrays can be linked to areas of rectangles.  Use arrays to explore the area of a rectangle. 6 rows of 2 squares. 6 x 2 = 6 equal groups of 2cm

Objective and Strategies	Concrete	Pictorial	Abstract
Multiplying by 10, 100 & 1000. (powers of 10)	Children will use place value counters, dienes & digit cards within place value frames to explore how numbers change as they are multiplied and divided by powers of 10.  When multiplying or dividing by a power of ten then number is placed in the place value frame. 1 x 10 = 10 10 ÷ 10 = 1 The children are taught that the number is increased or decreased 10x or 100x or 1000x and how the digits slide to the left (multiply) or right (divide) rather than learning the rule of 'adding a zero'. This is essential when working with decimal numbers as the children will understand how the value of each digit changes.	The same principle will be applied using pictorial representations. Children may be encouraged to draw their place value grid on a scrap piece of paper or maths jotter to visualise what is happening.  	Children will understand how numbers change when multiplied or divided by powers of 10. Given calculations like: 22 x 100 = 2,200 They will be confident in finding an answer using a range of informal strategies leading up to the ability to solve mentaly. **Mastery** Children will be able to mentally divide and multiply by powers of 10, including decimal numbers below 1. 0.89 x 10 = 8.9 0.95 x 100 = 95 0.26 ÷ 10 = 0.026

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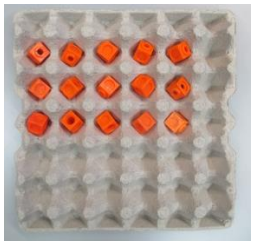
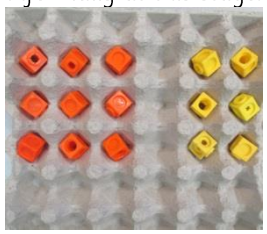
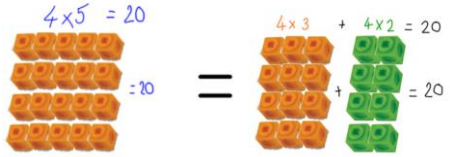
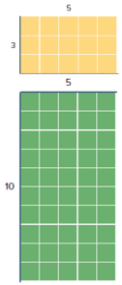
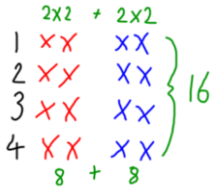
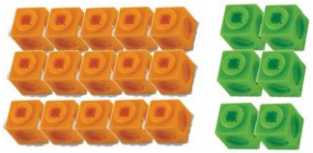
Multiplication - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Breaking down arrays ***(Distributive law)*** Solving mental calculations using the distributive law.	Breaking down arrays. Children can make an array for a multiplcation table. **E.g. 3 x 5** An egg box can be used to represent the array 3 x 5  This array can be broken down and the distributive law can be introduced informally at this stage.  By representing part of this array with a different colour, children can begin to see that the amount stays the same. e.g 4 groups of 5 = 5 + 5 + 5 + 5 = 20 = 4 x 3 = 3 + 3 + 3 + 3 = 12 **add** 4 x 2 = 2 + 2 + 2 + 2 = 8 **12 + 8 = 20**  This visual aid can be explored as a method of solving trickier calulations quickly. e.g. 7 x 3 I know 5 x 3 = 3, 6, 9, 12, **15** or 5, 10, **15** I know 2 x 3 = 3, **6** or 2, 4, **6** **15 + 6 = 21**	This can also be explored visually through pictorial arrays and the arrays drawn by the children.  What does this calculation show? How many in total? Can you use this to draw a representation of 14 x 2?  This can be developed and can support trickier tables using tables you are confident with.  The relationship between certain tables can be explored by breaking them down (distributive law) 2s and 4s 4s and 8s 3s and 6s etc. 	**Mental Calculations** Children will have a secure understanding of the distributive law and will know that: (3x4) + (2 x 4) = 5 x 4 They may use this to help them solve trickier calculations: 15 x 3 = **(10 x 3) + (5 x 3)** **(30) + (15)** **45** Children will be taught to use their understanding of arrays and multiplication when calculating mental multiplication. **34 x 5** I know 10 x 5 = 50 so I know that 30 x 5 = (10x5) + (10x5) + (10x5) = 150 I know 4x5=20 so 34x5 = 150 + 20 = 170

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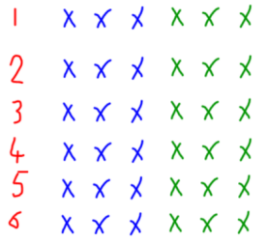
Multiplication - Calculation Policy

Objective and Strategies	Concrete & Pictorial	Abstract
3x table, 4x table, 8x table & 6x table.	Children use a variety of concrete objects to become fluent with the 3x, 4x, 8x tables. The 6 times table can be taught in the **summer term** and the link with the 3s can be made.  Use multiplication cards to see the tables visually. Left shows 3x2 (2 three times) and right shows 7x3 (or 3x7 – 3 7 times). The cards show patterns e.g. 5 3s and 2 3s. They are encouraged to look for examples of 'multiples' in every day life. How many arms and legs do three children have? There are four cars. How many wheels are altogether?  	Children use arrays to show groups of 3, 4, 8 & 6. Arrays can also be used to show the relationships between the tables. 6 x 3 = 18  6 x 6 = 36  The 6 times table is double the 3 times table. Bar models can be used to represent multiplication. 4 x 8 = 32 8 8 8 8
		Children are able to answer multiplcation statements fluently and quickly. 5 x 3 10 x 8 6 x 6 8 x 4 7 x 3 They understand and link the language of multiplication: Multiplied by – equal groups of – times by

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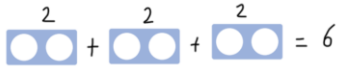
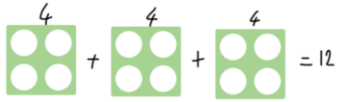
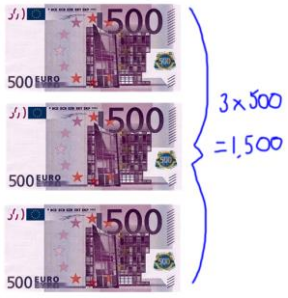
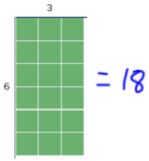
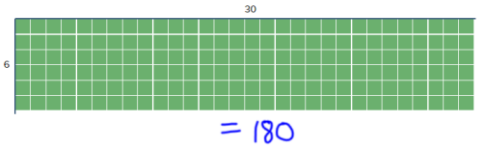
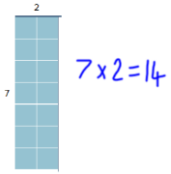
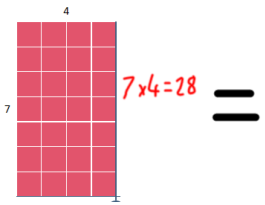
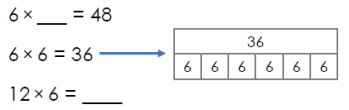
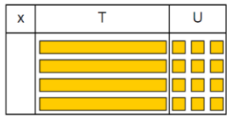
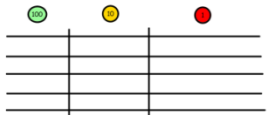
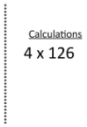
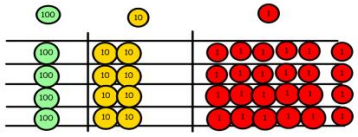
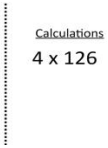
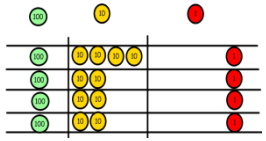
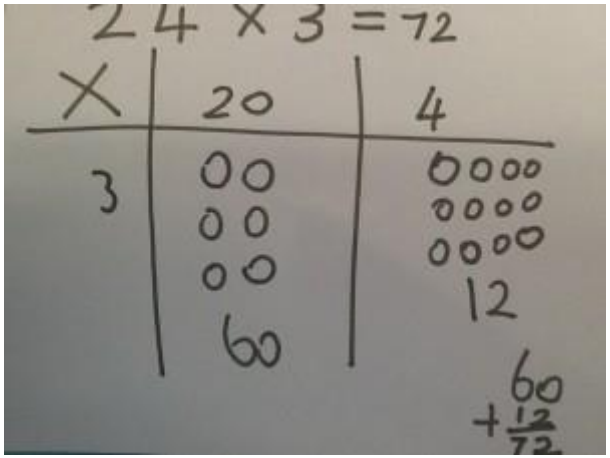
Multiplication - Calculation Policy

Objective and Strategies	Concrete	Pictorial	Abstract
Deriving facts from known tables.	<u>Linking Multiplication Tables</u> Building on the use of arrays, children can now begin to use a variety of resources to connect and explore the multiplication tabless.. They link the 2s to the 4s realising that the 4s are double. In the same way they relate the 3s and 6s and 5s and 10s   <u>Multiplication with multiples of 10</u>    Using resources, explore facts that can be derived. We know 3 x 5 How is that similar to 3 x 50 We know 3 x 5 so we also know 3 x 500	Show pictures of different arrays – ask the children to connect these together and spot patterns.  What is similar?  By drawing images themselves – children can begin to connect tables together.  The children can use numicon and other resources to make these links. The **distributive** law can be informally addressed to show these relationships.  	Children can confidently derive facts from multiplications tables. They reason and problem solve using these and can answer questions such as: I know... so...  **I know... so...** 7 x 5 = 35 70 x 5 = ___ ___ x 5 = 3500 **I know... so...** 2 x 6 = 12 4 x 6 = ___ ___ x 6 = 480

Objective and Strategies	Concrete	Pictorial	Abstract
The Grid Method	Make links with arrays when first introducing the grid method. 4 x 13 or 13 x 4. Children understand the commutative properties of multiplication so choose the array that will be most effective **(4 x 13)**  Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.   Fill each row with 126.   Add up each column, starting with the ones making any exchanges needed.   Then you have your answer.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. | | 10 | 8 | | --- | --- | --- | | 10 | 100 | 80 | | 3 | 30 | 24 | | X | 1000 | 300 | 40 | 2 | | --- | --- | --- | --- | --- | | 10 | 10000 | 3000 | 400 | 20 | | 8 | 8000 | 2400 | 320 | 16 |

<table>
<tr><th>Objective and Strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Scaling, Correspondence (ratio) & Combinations

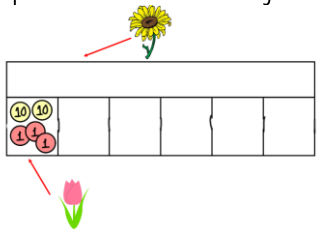
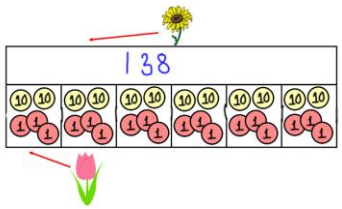
(context questions)

Scaling</td>
<td>
Scaling – When an object is increased (scaled up) based on an original measurement. Children will use cubes to explore this concept initially.

Jack has built a tower of 3 blocks. Sam build a tower 4 times as tall. How big is Sam's tower?
<table>
<tr><th>Jack's Tower</th><th>Sam's Tower</th></tr>
<tr><td>  </td><td>  </td></tr>
</table>

Bar moddeling can show this
<table>
<tr><td colspan="4">Sam's Tower = 12</td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>3</td></tr>
</table>

Larger number can be introduced when the concept is understood – Visual bar moddeling with place value counters can help show this.

A tulip is 23cm tall. A sunflower grows 6 times taller than the tulip. How tall is the sunflower?



</td>
<td>
<u>Scaling</u>

Children associate the 'times larger' with multiplication.

Sunflowers are 5 times taller than a tulip. Tulips are 35cm tall?
35 x 5

They draw on methods for multiplication to help them calculate.

Derrived facts?

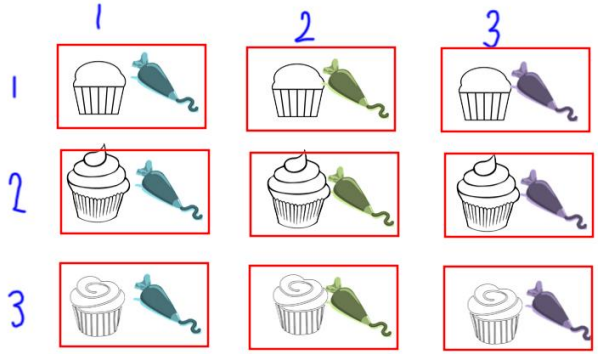
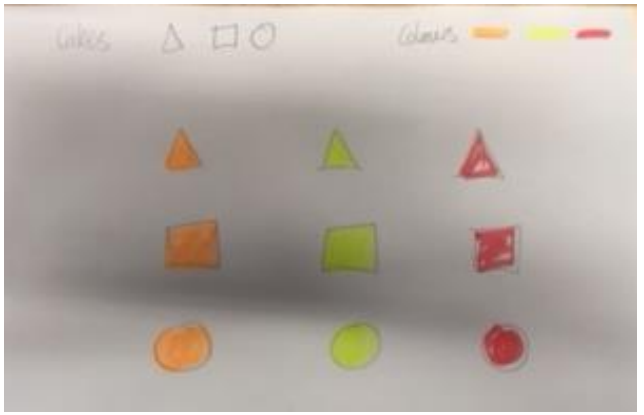
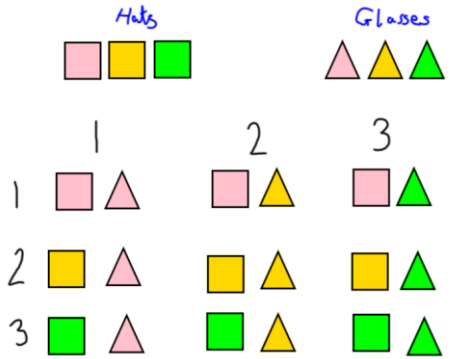
30 x 5 = (10 x 5) + (10 x 5) + (10 x 5) = 150
5 x 5 = 25

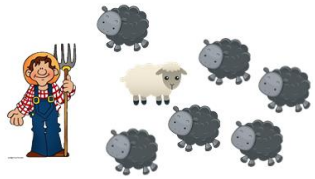
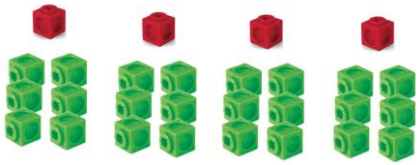
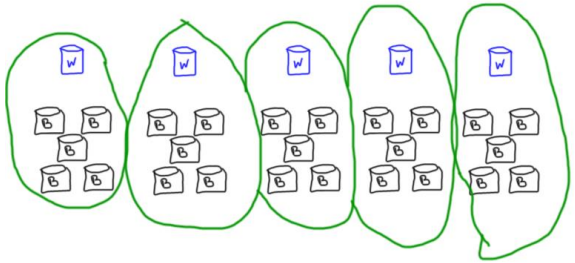
25 + 150 = 175.

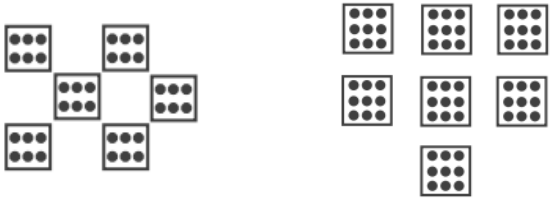
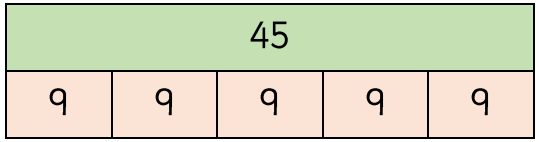
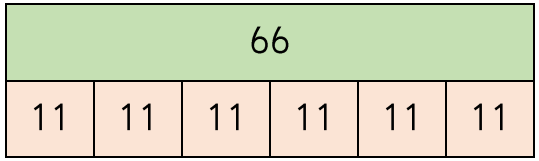
Bar modelling can be drawn to show this.
<table>
<tr><td colspan="5">Sunflower = 175</td></tr>
<tr><td>Tulips
35</td><td>Tulips
35</td><td>Tulips
35</td><td>Tulips
35</td><td>Tulips
35</td></tr>
</table>
</td>
<td>
<u>Scaling</u>

Children are presented with these written problems for a range of contexts. They draw upon effective methods to help them solve.

Concrete and pictorial models are relied upon less and less as children become secure with the concept of scaling.
</td>
</tr>
</table>

Objective and Strategies	Concrete	Pictorial	Abstract
Scaling, Correspondence (ratio) & Combinations (context questions) **Combinations**	**Combinations** Combination problems have multiplication at their heart but it can often be difficult for the children to see the relationship. Through using concrete resources, the problems can be explored and links can be made. Cakes are made in different shapes and are iced with different colours. The factory has three different shapes and three different coloured icing. How many combinations can be made?   By representing the problem visually, the children can see there are three combinations of colour per cake and three types of cake – 3 x 3 = 9 possible combinations.	**Combinations** Pictorial representations, less true to the original model could also be used.  There are 3 different coloured hats and 3 different colored glasses. How many different combinations of scarves and hats can be worn? 3 x 3 = 9  	**Combinations** Children are presented with these **written** problems for a range of contexts. They draw upon effective methods to help them solve. Concrete and pictorial models are relied upon less and less as children become secure with the concept of combination problems. *There are three breeds of male dog and four breeds of female dog. How many cross breed combinations could there be?* *3 x 4 = 12*

Objective and Strategies	Concrete	Pictorial	Abstract
Scaling, Correspondence (ratio) & Combinations (context questions) **Correspondence** (ratio)	**Ratio** The comparison of one set of objects in relation to another e.g. The ratio of tubs of white paint to black paint to make grey is 4:1. 4 tubs of white paint and 1 tub of black paint. When a farmer is buying sheep, he purchases 6 black sheep for every one white sheep. He purchases 4 white sheep. How many black sheep will there be?  Cubes can replace the sheep as a model for this ratio problem.  1 white sheep (red) for every 6 black sheep (green) We can see 4 red cubes and 4 x 6 green cubes. 6, 12, 18, **24** 1 + 1 + 1 + 1 = 4 6 + 6 + 6 + 6 = 24	**Ratio** Painter uses 1 tin of black paint for every 5 tins of white paint. Uses 5 tins of black paint. How many white tins does she need? Pictorial models could include drawing the problem out such as the example below.  1 1 1 1 1 = 5 5....5....5....5....5 = 25 Simutaneous Bar Models **White Paint** 5 1 1 1 1 1 **Black Paint** 25 5 5 5 5 5	**Ratio** Children are presented with these **written** problems for a range of contexts. They draw upon effective methods to help them solve. Concrete and pictorial models are relied upon less and less as children become secure with the concept of ratio. *Painter uses 1 tin of black paint for every 5 tins of white paint. Uses 5 tins of black paint. How many white tins does she need?* 5 x 1 = 5 = white paint 5 x 5 = 25 = black paint

Objective and Strategies	Concrete & Pictorial	Abstract
7x table, 9x table, 11x table & 12x table.	Children deepen their knowledge and fluency of the tables taught so far so they know confidently all table facts up to 12x12. The spend Y4 learning the 7, 9, 11 & 12 times table. Use a variety of concrete objects to become fluent with the 3x, 4x, 8x tables.  Multiplication cards can be used to support quick recall of the times tables. Children see the dots in the squares and then identify how many times this is shown. 6 x 6 and 7 x 9. Bar models can be used to represent multiplication. 5 x 9 = 45  11 x 6 = 66  	Children are able to answer multiplcation statements fluently and quickly. 9 x 3 11 x 8 12 x 6 8 x 9 7 x 7 They understand and link the language of multiplication: Multiplied by – equal groups of – times by

<table>
<tr><th>Objective and Strategies</th><th colspan="2">Concrete</th><th>Abstract</th></tr>
<tr><td>Column multiplication

(short)

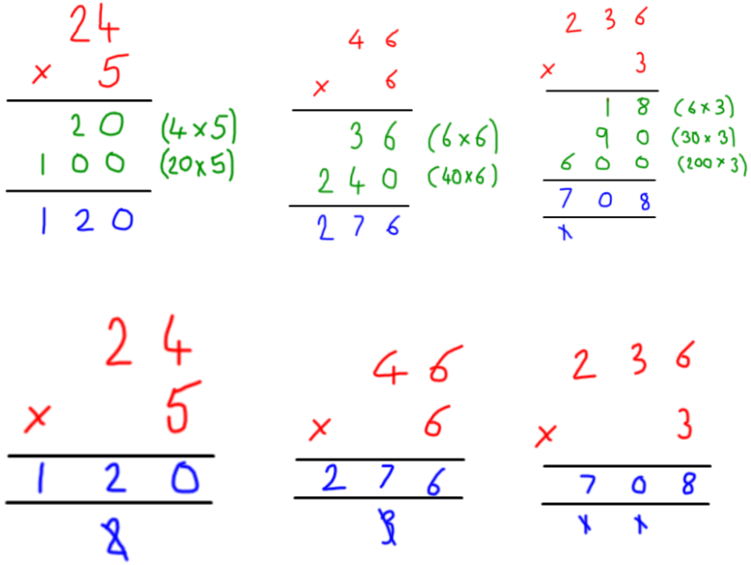
Two and three digits multiplied by a one digit number.</td><td colspan="2">Concrete and pictorial representations from the previous step can be drawn upon to support the children's understanding with this method.</td><td>In year 4, short multiplication is used to multiply two and three digit numbers by a one digit number only.

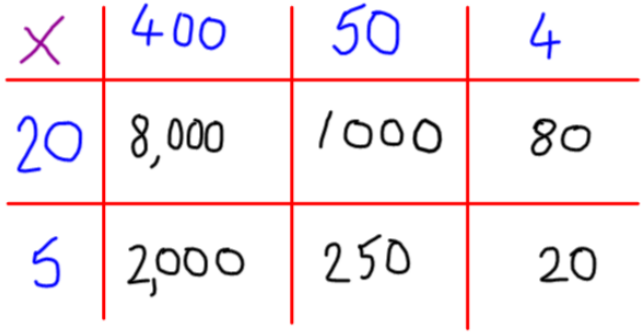
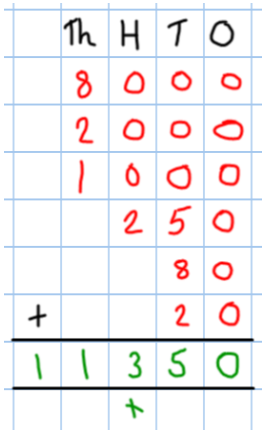
Long multiplication (2 digit x 2 digit) are introduced in the year 5 curriculum when this method is secure.

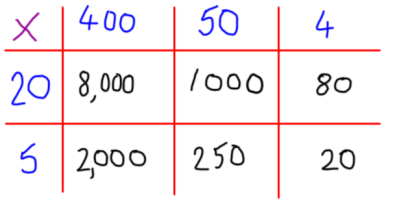
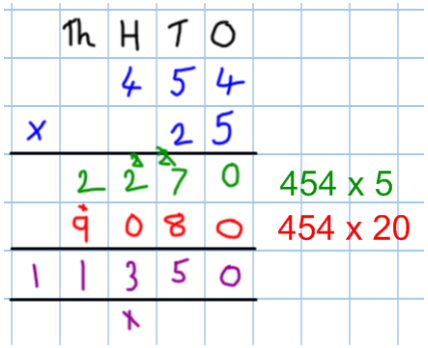
Column methods begin with expanding the process so that the children are aware of the steps.

A secure understanding of place value is essential.

Once secure, the children can move to a more compact method of calculating.

  </td></tr>
</table>

Objective and Strategies	Concrete	Pictorial	Abstract
Column multiplication (long) Up to four digits multiplied by a two digit number. **Grid Method**			In year 5, children are introduced to long multiplication. The grid can be used to first introduce the concept of multiplying by a two or three digit number. 454 x 25 becomes partitioned in the column. Each value is multiplied. The children can see that they need to multiply by 20 and then by 5 unlike the previous grid where the multiplication was by a single digit.  Each value is then added together to find the total.  This method can be accurate and will show the children the difference between multiplying by a one digit and two (or more) digit number. The children should grasp this quickly if their understanding of the grid is secure. Through discussion, the effectiveness of this method can be evaluated as there are many steps and margins for miscalculation.

Objective and Strategies	Concrete	Pictorial	Abstract
Column multiplication (long) Up to four digits multiplied by a two digit number. **Formal Method**			This grid method now becomes.  The calculation is set out in a grid and then both calculations are added together.  Children will be expected to progress to multiplying 4 digits by two digits using the long multiplication method as opposed to the long winded and less efficient grid method.  2148 x 34 Is set out using the long multiplication grid.

Objective and Strategies	Concrete	Pictorial	Abstract

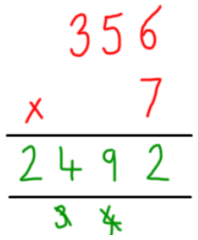
Mastery of Multiplication

When children have mastered the strategies for multiplication, activites and challenges (such as these examples) can be used to deepen, develop and test their understadning.

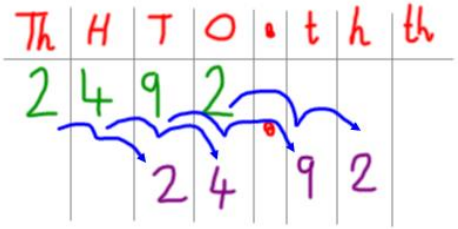
Multiplying decimals

3.56 x 7

Children may multiply 3.56 by 100 = 356 then calculate and divide by 100 to find the answer.



The answer 2492 can be divided by 100 to find the original answer.



Multiplying by 10 or 100

15 x ____ = 1515

Can the children spot 15 x 100 = 1500?

1515 is 15 more than 1500 which is 1 x 15.

So the answer must be:

15 x 101 = 1515

If you know... can you work out?

16 x 24 = 384

How can you use this fact to work out 16 x 22?

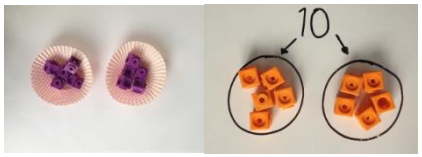
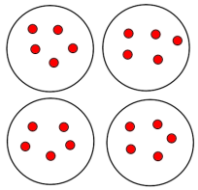
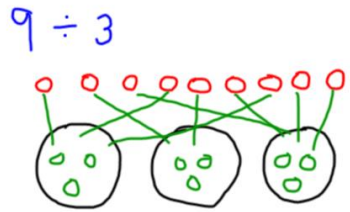
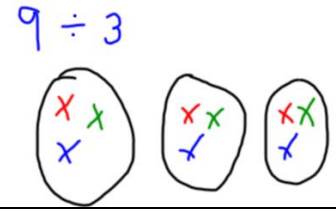
22 is 2 less than 24.

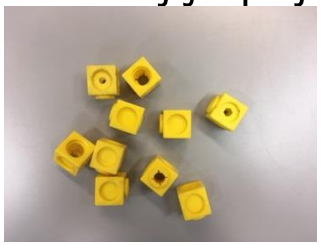
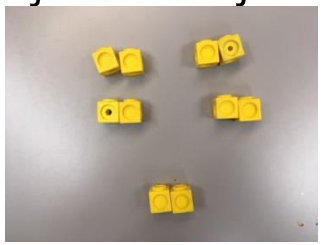
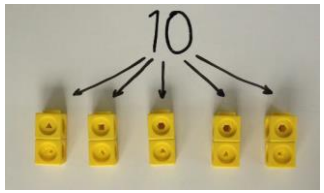
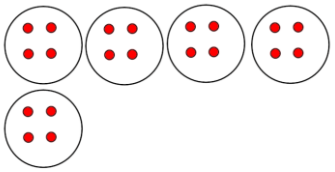
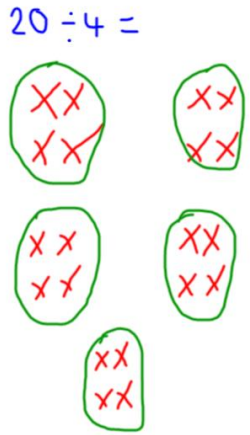
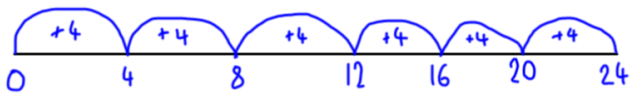
16 x 24 = (16 x 22) + (16 x 2)

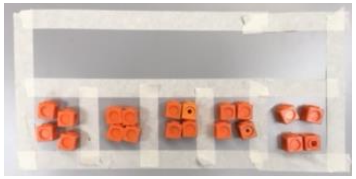
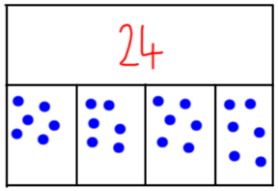
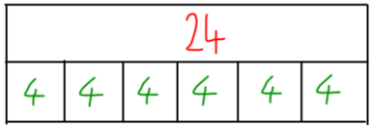
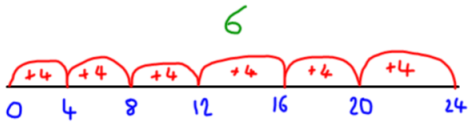
16 x 2 = 32

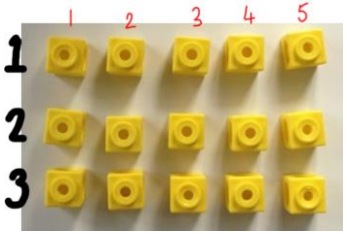
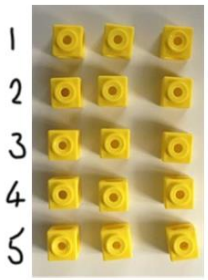
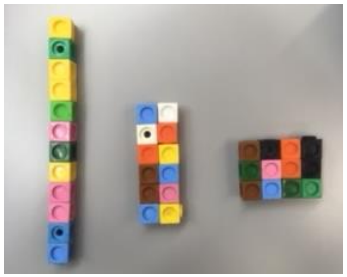
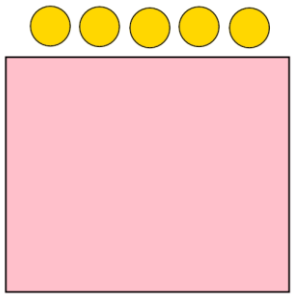
384 – 22 = 352

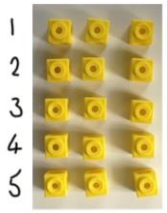
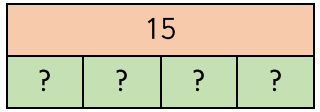
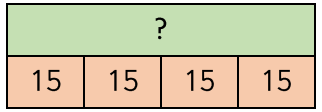
Division

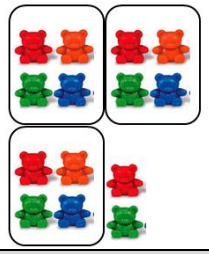
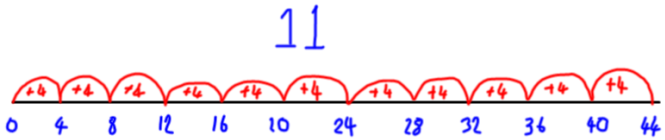
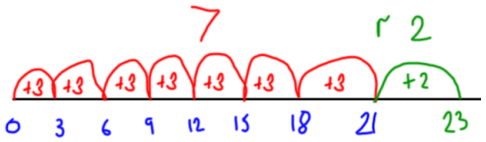
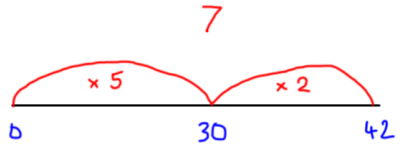
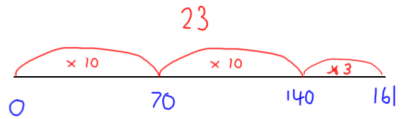
Objective and Strategies	Concrete	Pictorial	Abstract
Division as **sharing** objects into **equal groups**	The children will explore the concept of division being the sharing of objects. Contexts will be used to support the understanding of this concept. 10 sweets are shared between 2 friends. How many sweets do they get each?  Models and resources are used to support this concept  When sharing the **number of groups** is fixed and we are counting how many in each group. E.g. 20 ÷ 4 There will be 4 groups and the 20 will be shared into these groups  There are 4 groups and there are 5 in each of the groups.	Different pictorial representations may be used alongside the concrete. Initially, drawing the original amount then matching to groups.  Progressing to holding the amount being shared in your head and counting this out into the groups being shared into.  "1,2,3" "4,5,6" "7,8,9" Three in each group.	Children will still use a range of informal strategies (sharing circles) to help them visualise and calculate. The informal calculation **must** be shown alongside a context. Share 9 buns between three people. 9 ÷ 3 = 3 Seeing division recorded this way will allow the children to make links with the practical application of sharing and the calculation in its written form.

Objective and Strategies	Concrete	Pictorial	Abstract
Division as **grouping** objects.	Most early division experience will be through sharing. Children will be confident with the idea of sharing objects and need to develop the concept of division as grouping alongside this. **How many groups of 2 can you make out of 10?**   I have 10 and I can make 5 groups of 2.  When grouping we need to think about this calculation (20 ÷ 4 =) as how many 4s in 16 as opposed to 16 shared into 4 equal groups. The amount is **fixed** and we are counting the **amount of groups.**  There are 5 groups of 4. 20 ÷ 4 would be 5	These groups may be represented pictorially as above with circles. The examples below represent methods that could be used. The children will work initially with **tables they know**. *The children group the crosses in 4s and count as they draw these – 1,2,3,4 5,6,7,8 9,10,11,12 etc…*  The concept of grouping can be further developed (Y2) through the use of a number line. 24 ÷ 4 = 	Children will still use a range of informal strategies to help them visualise and calculate. The informal calculation **may** be shown alongside a context. Share 20 buns between 5 people. 20 ÷ 5 = 4 Division calculations will be using the multiplication tables they know (2, 5, 10).

Objective and Strategies	Concrete	Pictorial	Abstract
Contexts for division.	Contexts are used continually in the early stages of division. Children will develop their use of a range of concrete and pictorial models to support their conceptual understanding. **Sharing** 16 sweets are shared between 4 children. How many sweets does each child receive?   Visual Bar Model  **Grouping** Children get given sweets at the school fair. Each child gets 4 sweets, there are 20 sweets. How many children will get 4 sweets?  	**Sharing** A zoo keeper has 24 penguins and wants to separate them into four enclosures. How many penguins will be in each enclosure? The bar model can be used to help the children see the number being shared.   **Grouping** A zookeeper has 24 penguins. He wants each enclosure to house 4 penguins. How many enclosures will he need? This bar model can be used to represent 4s. There are 6 groups. This can also be solved using a number line.  	Children will understand the operation they are solving when calculating these context questions. $24 \div 4 = 6$

Objective and Strategies	Concrete	Pictorial	Abstract
Division with arrays	The link between multiplication and division can be explored through arrays. 15 ÷ 3 = 5 Share 15 into 3 equal rows.  15 ÷ 5 = 3 Share 15 into 5 equal rows.  There are 12 cubes. How can you arrange this into an array?  12 rows of 1 = 12 ÷ 12 = 1 6 rows of 2 = 12 ÷ 6 = 2 3 rows of 4 = 12 ÷ 3 = 4	There are 12 oranges in a shop. The shop keeper shares these into 4 display bowls. How many oranges will be in each bowl?  Deepen the concept of arrays by showing the array covered up. *There are 25 circles. How many rows will there be?* 25 ÷ 5 = 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Objective and Strategies	Concrete	Pictorial	Abstract
Language of division		Dividend, Divisor, Quotient 30 ÷ 5 = 6	
Deepening division concepts		Children need to develop their early understanding of division so that they will become increasingly fluent when beginning to use written calculation methods. The concept is explored e.g. 10 ÷ 5 = 2 is not the same as 5 ÷ 10 = 2 Arrays can be used to show the importance of the order of numbers. Division, unlike multiplication, is not commutative and cannot be calculated in any order. However, as explored before, arrays can show the relationship between the divisor and the quotient when swapped. 15 ÷ 5 = 3  15 ÷ 3 = 5  Bar models can be used to get the children thinking.  This calculation will require 15 to be divided by 4 15 ÷ 4 = ☐  This calculation shows 15 four times. This calculation required the inverse, multiplication. ☐ ÷ 4 = 15	

Objective and Strategies	Concrete	Pictorial	Abstract
Remainders with division	14 ÷ 3 = 	Pictorial representations are introduces along side the manipulatives. The concept of 'equal' is deepended and if an amount doesn't fit, it becomes a remainder.  	Children show a remainder using 'r' when writing the written calculation. **29 ÷ 8 = 3r5**
Early Written Methods *Number line*		Children begin to perform divisions by counting up in the divisor using a number line. **44 ÷ 4 =**  Remainders can be explored with the number line. **23 ÷ 3 =**  More efficient steps are encouraged as children become confident with their knowledge of multiplication tables. **42 ÷ 6 =**  **161 ÷ 7 =**  	The number line introduces the children to the concept of partitioning numbers in different ways to be able to calculate. e.g. 84 ÷ 4 = 21 80 + 4 = 80 ÷ 4 = 20 4÷4 = 1 84 ÷ 7 = 12 70 + 14 70 ÷ 7 = 10 14 ÷ 7 = 2

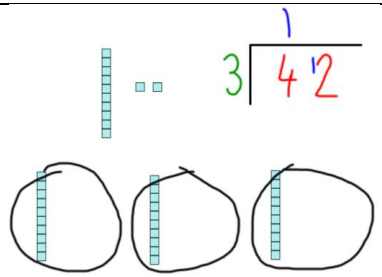
Objective and Strategies	Concrete	Pictorial	Abstract
Early Written Methods *Short Division with manipulatives (1)*	Short division is introduced using manipulatives to support. At this point remainders are not intoduced until the concept is secure and calculations work with tables the children are learning and are confident in. Y4 children are expected to work with remainders so this concept can be introduced depending on the children's level of confidence.		

42 ÷ 3 =

1

The dividend is made using dienes or place value counters.

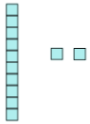
2



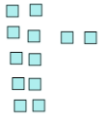
This can be physically shared into three groups.

Each group gets 1 and there are 12 remaining. Carry the 10 into the next column.

3

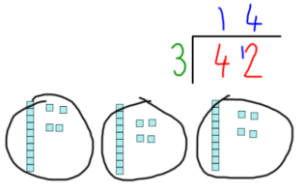


The ten in 12 is exchanged for 10 ones



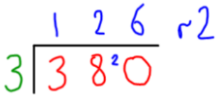
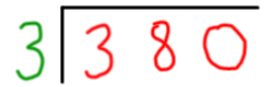
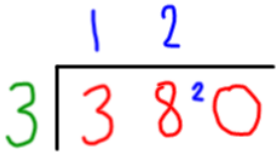
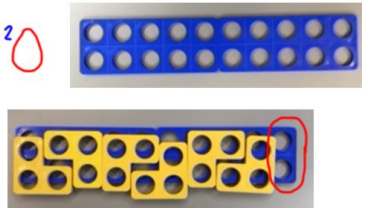
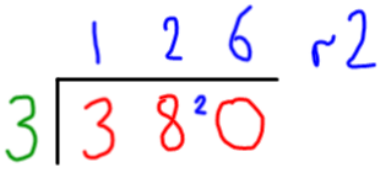
Then the 12 ones can be shared out into the three groups.

4



3 goes into 12 4 times.

The answer is 14

Objective and Strategies	Concrete	Pictorial	Abstract
Early Written Methods *Short Division with manipulatives (2)*	As children become increasingly fluent using manipulatives to support with the method, larger numbers can be introduced. This will test the children's understanding but to be quick and efficient, it will require them to use and apply multiplication table facts and grouping rather than using the manipulatives for every stage. **380 ÷ 3 =**		As the procedure of short division develops, the children are taught that they are partitioning the number e.g.  300, 60 and 18 are all multiples of 3 and the remainder 2 isn't 300 = 100 x 3 60 = 20 x 3 18 = 6 x 3 2 380 Children will become increasingly fluent with this method and will rely less upon concrete resources.
	1 	How many 3s in 3 – 1 How many 3s in 8 – 2 with 2 remaining	
	2 	This is written into the short division method. When looking at the 20 this could cause confusion. We can't use inverse facts (grouping) e.g. 20 ÷ 3 or __ x 3 = 20 as there isn't one.	
	3 	By representing the 20 using numicon or other manipulatives, the concept of remainders can be secured. 3 goes into 20 - 6 times with 2 remainder.	
	4 	This can be reflected in the written method.	

Objective and Strategies	Concrete	Pictorial	Abstract

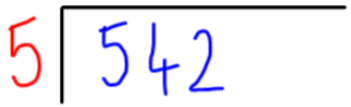
The children will become increasingly confident with this method and will progress towards changing the remainder into a decimal.

542 ÷ 5 =

Early Written Methods

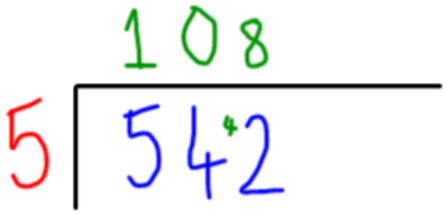
Short Division – Interpreting Remainders as decimals.

1



The calculation can be written.

2

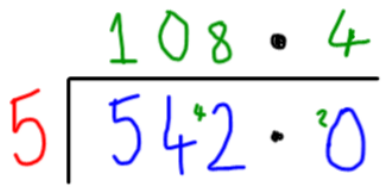


Begin by thinking how many times does 5 go into 500? 100x5 = 500 because I know 1 x 5 = 5.

Continue with the method until left with a remainder.

I have **2 remaining.** Continue the calculation by extending the whole into the decimal place value.

3



Now continue the calculation.

Mastery challenges could include questioning the children about reccuring numbers when dividing.

Objective and Strategies	Concrete	Pictorial	Abstract

Mastery of Division

As the children become confident with the concept and methods for division, their understanding is taught and expanded. Rather than being allowed to work with larger numbers, purposeful questions are written to allow the children to explain and reason with the skills they have acquired.

A taxi can fit 4 passengers. There are 67 guests at a wedding. How many taxis will be needed to transport the 67 people home?

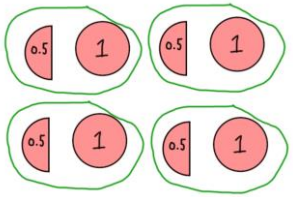
Applying their fluency will give them the answer of:

$$4 \overline{)67} \quad 16 \text{ r } 3$$

The children will need to realise that the remainder 3 will also need a taxi and round the answer to 17 to answer the word problem.

6 ÷ 1.5 =

The children will need to understand that they will have to use grouping to solve this task. A formal method would be more difficult.



6			
1.5	1.5	1.5	1.5

Inverse operations test the children understanding of multiplication.

□ ÷ 4 = 6

Inverse with remainders further test this.

□ ÷ 4 = 7 r 3

The r3 wasn't a multiple of 4.
So the inverse would look like this:
7 x 4 = 28
Then add the remainder.
28 + 3 = 31

Interpreting remainders as fractions

67 ÷ 4 =

$$4 \overline{)67} \quad 16 \text{ r } 3$$

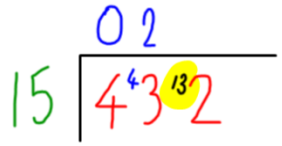
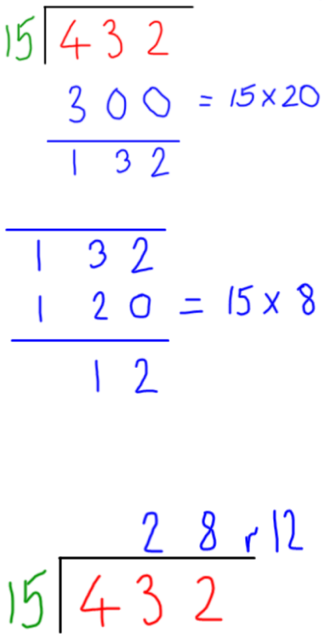
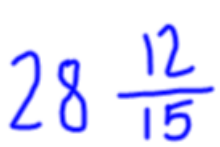
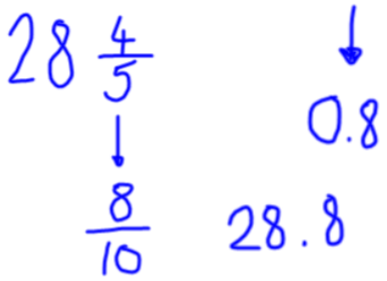
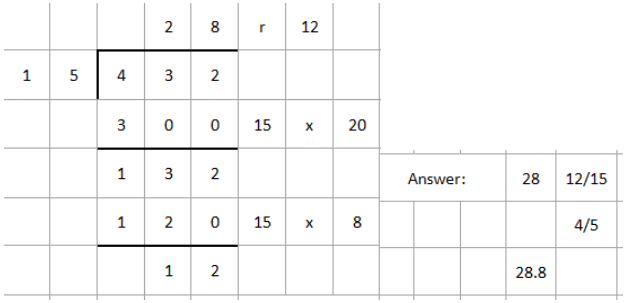
The remainder 3 is represented as

$$\frac{3}{4}$$

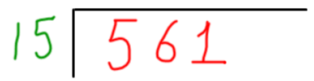
3 being the remainder and 4 being the divisor.

This is further developed with the understanding of decimals. Children with know the decimal value for 1/2, 1/4, 1/10 and ¾ and the equivalent fractions for ½ e.g. 2/4.

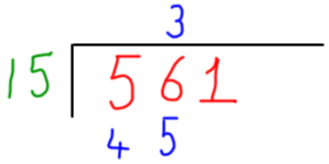
¾ becomes 0.75 so 16r3 becomes 16.75

Objective and Strategies	Concrete	Pictorial	Abstract
Long Division *Chunking*		Children will be confident with the method but may find difficulties initially when dividing by a two digit number. e.g. 432 ÷ 15  The children may get to this stage and find a remainder that is a two digit number. This may cause them to struggle. A long division procedure is used.  Identify multiples of 10 first. 15 x 10 is 150 15 x 20 is 300 Subtract the 300 what is left 132 Continue working down. What is the next largest multiple you can find. 15 x 5 is 75. 15 x 3 is 45 15 x 8 = 120 with 12 remaining. The answer can be written in.	 The remainder can be interpreted as a fraction. This can be simplified if the children are confident. 15 ÷ 3 = 5 12 ÷ 3 = 4  In books: 

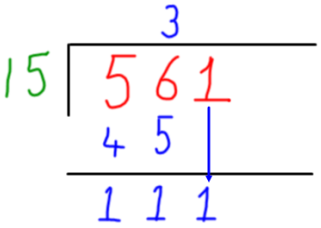
Objective and Strategies	Concrete	Pictorial	Abstract
Long Division *Traditional Method*		When the method is secured, the children can work towards a more efficient method of calculating.	



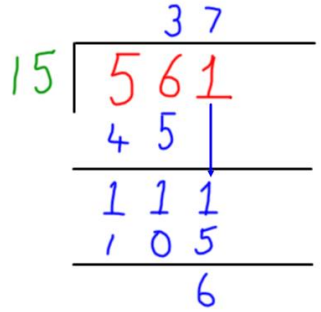
Begin with the written method. How many times does 15 go into 5? None. What about 56?
3 x 15 = 45 so 3 times with 11 remaining.



Subtract the 45 from 56. See the remainder and the 1 is pulled down.
How many 15s in 111?



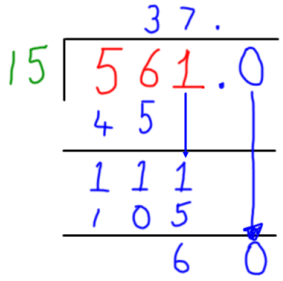
7 x 15 = 105



Subtract the 105 and you have 6 remaining.

This could cause us a problem as we have no more digits left.

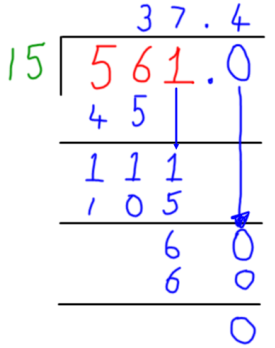
We need to go into decimal values, so adjust the 561 and bring 0 tenths down.



How many times does 15 go into 60? 4 times.

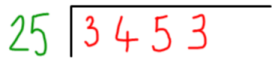
How many times does 15 go into 6? 0.4.

This is reflected in the calculation.

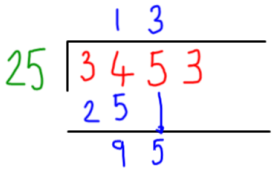


The calculation is complete and the answer is 37.4

Objective and Strategies	Concrete	Pictorial	Abstract
Long Division *Larger numbers*		As children become fluent with the method, they can apply their understanding towards larger numbers	

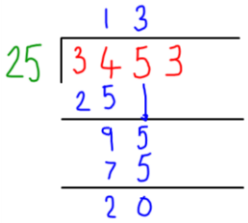


Again, begin with the written method. How many times does 25 go into 3? None. What about 34?
1 x 25 = 25. 34 – 25 = 9

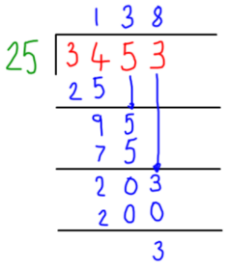


Bring the 5 down.

How many times can 25 go into 95?
3 x 25 = 75

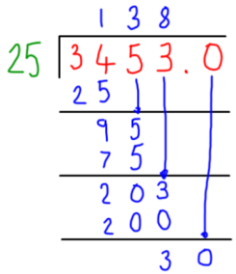


There are 20 left over. How many times does 25 go into 20? None so bring the 3 down.
How many times will 25 go into 203?
8 x 25 = 200
203 – 200 = 3 left over.



There are no more whole number digits left and we still have three remainding.

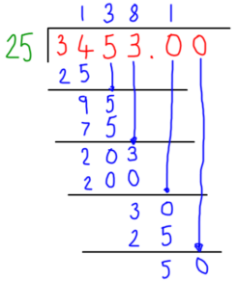
Add in the decimal zero and bring this down.



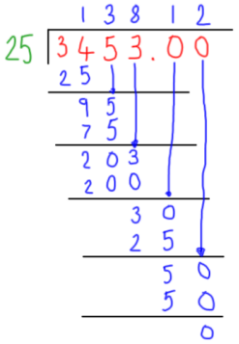
How many times can 25 go into 30?

Once with 5 remaining.

Repeat the process of including a decimal zero and bring this down. Complete the calculation by identifying how many times 25 can be placed into 50.



2



The calculation is completed and the answer is 138.12